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The Planetary Equatorium of Jamshīd Ghiyāth al-Dīn al-Kāshī

Princeton Oriental Studies

Volume 18

THE PLANETARY EQUATORIUM

OF JAMSHĪD GHIYĀTH AL-DĪN AL-KĀSHĪ

(d. 1429)

An Edition of the Anonymous Persian
Manuscript 75[44b] in the Garrett Collection
at Princeton University
Being a Description of Two Computing Instruments

The Plate of Heavens
and the Plate of Conjunctions

WITH TRANSLATION AND COMMENTARY BY

E. S. KENNEDY

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Jan/Kev

1960

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To

my foster mother

Annie E. Kennedy

PREFACE

This book presents, with translation and commentary, the text of a Persian manuscript in the Garrett Collection at Princeton University. The manuscript describes the construction and use of two astronomical computing instruments invented by the fifteenth century Iranian scientist Jamshīd ibn Masʿūd ibn Mahmūd, Ghiyāth al-Dīn al-Kāshī (or al-Kāshānī). Our fund of knowledge concerning the work of this individual has been materially increased in recent years, particularly by the studies of the late Paul Luckey in Germany, and by the fruitful collaboration of B.A. Rosenfeld and A.P. Yushkevich in Russia. The reader will find their publications listed in the bibliography at the end of this volume.

Little enough has been known about the life of Kāshī (as we shall call him hereafter), and we have been able to supplement that little, principally by examining the introductions and colophons of Kāshī manuscripts. The results are assembled in the biographical sketch immediately following the table of contents.

The succeeding section recites the relations between our anonymous Persian text and the two versions of a book by Kāshī himself on which it is based.

There follows the facsimile text and translation, arranged for immediate reference with corresponding pages and lines opposite each other. After this comes the commentary to the text. In general, topics are treated in the order in which they appear in the manuscript. Where it has been found convenient to digress from this order, a statement has been inserted indicating the proper section. When sections are related to a particular passage in the text, this passage has been specified in parentheses following the section title. All such references to the text give folio and line, separated by a colon.

It is assumed that the reader is familiar with the leading concepts of Ptolemaic astronomy, of which [41] and [42] contain

readily available expositions. Here and in the sequel, numbers enclosed in square brackets refer to items in the bibliography at the end of the book.

The bulk of the manuscript is taken up with an instrument called by Kāshī Ṭabaq al-Manāṭiq, which we translate as "Plate (or Tray) of Heavens." It is an example of the class of devices known as equatoria, analogue computers on which the various Ptolemaic planetary configurations were laid out to scale. They thus yield solutions to such problems as that of finding the true longitude of a planet at a given time. Kāshī's equatorium is only one element in a tradition stretching through sixteen centuries in time, and in space from Western Europe to Central Asia via North Africa and the Near East. The story of the other instruments, and the Plate of Heavens' place therein, has been delineated admirably by D.J. Price in his publication [42] of the Chaucerian equatorium - there is no need to repeat it here.

The second instrument, the "Plate of Conjunctions" (Lawḥ al-Ittiṣālāt), is a simple device for performing a linear interpolation.

A number of the problems raised by the manuscript have been dealt with in a preliminary way in papers published over a period of years. This is an attempt simultaneously to dispose of the remaining problems, to revise, complete, and correct previous solutions, and to put before the public the primary source on which they are based. In addition to presenting the philologians with an opportunity to pick apart the work of a translator whose formal training has been in mathematics, this will serve to present intact a medieval scientific work, a very small link in the chain which leads from the tallystick to the electronic computer. For the manifold shortcomings of the result the editor, like the anonymous author of the manus-

cript, craves correction "with the musk-dripping pen" of forebearance.

Acknowledgments

In thus bringing to a close a task, the working out of which has been a source of much satisfaction over a long time, it is pleasant indeed to set down the names of some of the many friends who have been helpful. Professor Yahya Armajani called to my attention the equatorium manuscript in the first place. In the office of the late Professor George Sarton I was privileged to associate with Dr. Alexander Pogo, and from him gained my first real insight into planetary motions. Professors N. Seifpur Fatemi and Jalal Homa'i, and Dr. Iraj Dehqan answered many questions connected with the reading of the Persian text. The commentary has been improved as a result of many discussions with Professor Derek Price, whose book on the Chaucer equatorium will continue to maintain itself as the model of publications of this genre.

The contributions acknowledged above, important as they are, can be regarded as peripheral and as not involving the individuals in errors which the book may contain. It remains to name another associate, one who cannot escape his portion of responsibility for whatever merit this work has. To the counsel and example of O. Neugebauer, best of friends and keenest of critics, I owe such competence as I have acquired in the history of the exact sciences.

The work was made possible by the Rockefeller Foundation, Brown University, the Institute for Advanced Study and a Fulbright grant. The Princeton University Department of Oriental Languages and Literatures and the Princeton Univer-

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Copy for the photo-offset reproduction was typed by Mrs. Kawthar A. Shomar with her customary accuracy, elegance, and dispatch.

E.S.K.

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INTRODUCTION

Biographical Material on the Inventor

Our earliest fixed point in Kāshī's life is 2 June, 1406 (12 Dhū al-Hijjah, 808). On this date he observed in Kāshān, his home town in central Iran, the first of a series of three lunar eclipses (see [23]*, f.4r.)

He is the author of an Arabic treatise on the sizes and distances of the heavenly bodies, called Sullam al-samā' [22]. Most of the extant copies of this work are undated, but in [34], p.50, Krause reports that one of the Istanbul copies claims the book was finished on 1 March, 1407 (21 Ramadān, 809). This is confirmed by Ṭabāṭabā'ī ([52], p.23) on the basis of a copy at the Shrine Library in Meshed, Iran. Since this date falls during the run of eclipse observations, it follows that the treatise must have been written in Kāshān. It is dedicated to a certain wazīr designated in the manuscript only as Kamāl al-Dīn Maḥmūd. Search through general histories of the period has failed thus far to produce any information whatsoever as to the identity, jurisdiction, or political affiliation of this individual. A certain Maulā Kamāl al-Dīn Maḥmūd al-Sāghirjī mentioned by Khwāndamīr ([33], vol.3, p.513) fulfills the requirements as to name and rank, but cannot have been contemporary with Kāshī. It is probably to this individual that Ṭabāṭabā'ī ([52], p.23) refers.

Sometime during the year 816 A.H. (1413/14) Kāshī completed the Khāqānī Zīj [23], written in Persian and the first of his two major works. In the introduction to it he complains that he had been working on astronomical problems for a long period, living in penury in the cities of ʿIrāq (doubtless ʿIrāq-i ʿAjamī, Persian ʿIrāq), and most of the time in Kāshān. Having undertaken the composition of a zīj, he would have been unable

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* Numbers in square brackets refer to items in the bibliography.

to complete it except for the timely beneficence of the prince Ulugh Beg, he says, and to him he dedicated the finished work. The fact that the longitude of Shīrāz was taken as base for the mean motion tables (see Section 18 in the commentary below) does not prove that he worked there part of the time, although he may have done so. The place had been a center of astronomical activity on and off for many centuries, and Kāshī may simply have chosen as base a location better known than Kāshān. From a remark in his Miftāḥ ([47], p.176) it is clear that he was in the neighboring city of Isfahān at one time or another.

Next comes a very short (seven page) Persian treatise [20] on astronomical instruments written in January 1416 (Dhū al-Qaʿda, 818) and dedicated to a Sultan Iskandar.

Soon after this, on 10 February, 1416 (10 Dhū al-Hijjah, 818), and in Kāshān, the first version of the Nuzhat al-Hadā'iq was completed. The book is Kāshī's own description of the equatorium he invented. It names no patron. When next heard from, Kāshī has joined the group of scientists at the Samarqand court of Ulugh Beg. There he stayed for the rest of his life.

His career commenced during the long reign of Tamerlane. When the latter died in 1405 he was eventually succeeded by his son Shāhrukh, whose rule outlasted Kāshī's life span, and who throughout evidently retained some sort of hegemony over all of Iran. During this time Shāhrukh's son Ulugh Beg was the ruler of Samarqand, and eventually as head of the Timurid dynasty survived his father for a short time.

The Iskandar referred to above can hardly have been any other than the son of Qara Yūsuf ([12], p.127) second ruler of the Black Sheep Turcoman dynasty which established itself mainly in Azarbaijan and Mesopotamia, encroaching eventually into Fārs. Iskandar was twice defeated by Shāhrukh. Like Ulugh Beg, he achieved primacy in his dynasty only long after the death of Kāshī.

INTRODUCTION

The curious spectacle of a scientist dedicating successive writings to minor potentates in rival dynasties invites speculation. It is tempting to liken his actions to those of a modern scholar alternately wooing one and another of the affluent learned foundations in hopes of an ever more princely grant. Kāshī's short treatise on astronomical instruments [20] contains little beyond what must have been common knowledge to any competent astronomer of the day, and its composition can have cost him only the time required to write it down. He may have turned it out and dedicated it to Iskandar in order to counteract the effect of the earlier dedication to a Timurid. But ignorance of the details of political vicissitudes makes further conjectures unprofitable.

In 1417 Ulugh Beg commenced building in Samarqand a madrasah ([16], p.54), a school to house students of theology and the sciences. This impressive tiled structure is still admired by tourists from all over the world. Shortly after its completion the construction of the observatory was begun.

It was during these operations that Kāshī arrived; we do not know exactly when. Several sources (e.g. [33], vol.4, p.21) have him accompanied by a fellow-townsman and astronomer, Muʿīn al-Dīn al-Kāshī, according to Ṭabāṭabā'ī ([52], p.6), a nephew. From this period also is a document of the greatest interest, a letter from Kāshī to his father in Kāshān. It has been published (as [53]) in the original Persian, collated and annotated by M. Ṭabāṭabā'ī from two manuscripts, one of which is in the library of the Madrasah Sepahsālār in Tehrān. The letter merits publication in facsimile with a translation into some European language. Pending this we outline its contents below.

The epistle begins with a quotation from the Qur'ān indicative of filial piety, and a statement that the writer has been too busy with the observatory to do anything else. He writes that the sultan is an extremely well educated man, in the Qur'ān,

in Arabic grammar, in logic, and the mathematical sciences. As an illustration of the latter he tells how the king, while on horseback, once computed in his head a solar position correct to minutes of arc. Kāshī then goes on to describe how, upon his arrival at Samarqand, he was put through his paces by the sixty or seventy other mathematicians and astronomers in attendance there. He gives as examples four of the problems propounded to him. The first involved a method of determining the projections of 1022 fixed stars on the rete of an astrolabe one cubit in diameter. The second required the laying out of the hour lines on an oblique wall for the shadow cast by a certain gnomon. The third problem demanded the construction of a hole in a wall, of such a nature that it would admit the sun's light at, and only at, the time of evening prayer, the time to be that determined by the rule of Abū Ḥanīfah. Lastly, he was asked to find the radius, in degrees of arc on the earth's surface, of the true horizon of a man whose height is three and a half cubits. All these and others, says Kāshī, which had baffled the best minds of the entourage, he solved with ease, thus quickly gaining intellectual paramountcy among them.

He held a low opinion of the rest of the sultan's scientific staff in general, in spite of the fact that Ulugh Beg's astronomical bent had stimulated the study of mathematics in Samarqand for the past ten years. The only one who gave him any competition at all was Qāḍīzādah al-Rūmī ([51], p.174), and Kāshī recounts in detail two occasions on which he worsted this individual. One of these was brought on by Qāḍīzādah, who had been expounding the famous zīj of al-Bīrūnī, the Masudic Canon [5], when he ran into difficulties with the proof of a theorem. Using the immemorial gambit of any mathematics teacher when faced with a like situation, he told the class, which included Ulugh Beg himself, that there must be a

fault in the text, he had best compare it with a better copy elsewhere. After two days he was still stuck at the same place, when Kāshī just happened to turn up, explained the proof off-hand, and showed that the manuscript was correct.

In spite of this, and other exhibitions of tact and erudition on the part of Kāshī, he assures his father that relations between the two of them are of the most amicable. He agrees that Qādīzādah is the only one of the lot who knows much about the Almagest, although he is deficient in observational technique. The views of Qādīzādah on the matter have not come down to us, but it may or may not be of significance that in the prolegomena to Ulugh Beg's zīj ([54], p.5) Qādīzādah is the first of the two to receive honorable mention and extravagant praise.

Kāshī uses up a good deal of space telling his father about the ignorance of a certain Badr al-Dīn. This man, he writes, having been through a few propositions of Euclid which he is unable to apply, is like one who knows several rules of (Arabic) grammar but can write no Arabic. He states further that Badr al-Dīn is a liar. Kāshī's motive in reporting on this person is not clear.

The letter describes the status of the work in progress on the construction of instruments for the observatory. It closes with a detailed explanation intended to make clear to the layman why the taking of observations for a complete set of planetary parameters is a long process, and cannot be completed in as short a time as a year, or anything like it.

In the middle of Sha'bān, 827 (July, 1424), Kāshī finished his unprecedentedly precise π-determination, al-Risālat al-muhitīyah. Written in Arabic, this masterpiece of computational technique has been published in German [36] and Russian [47] editions, both excellent. It has no dedicatory passage.

From the same month two lunar years later (June, 1426)

dates the Samarqand rescension of the Nuzhah, and on 3 Jumādī I, 830 (2 March, 1427), Kāshī completed his second major work, the Miftāh. It is dedicated to Ulugh Beg, and has recently been published [47] with Arabic text, Russian translation, and commentary, by Rosenfeld and Yushkevich. Studies based on it are [35], [37], [44], and [7]. In the introduction to the Miftāh Kāshī gives a partial list of his own works. Other than titles already mentioned above, the following appear:

Risālat al-watar w'al-jaib, also known as Risālah fī istikhrāj jaib darajat wāhidah, is apparently extant both in a lithographed edition printed in Tehran in 1889, and in manuscript ([51], p.174). Marginal notes in [23], f.32r, and the lithograph edition of [17], p.3, state that the treatise was incomplete when Kāshī died and that it was finished by Qādīzādah. A copy of the lithographed collection of which it is a part is in the Parliament (Majlis) Library in Tehran. It describes an elegant iterative method of computing the sine of one degree to any required accuracy. No translation of this risālah has been published, although a commentary on it is available in French [54] and in Russian [47] translations, and a considerable literature in European languages has accumulated about it (see, e.g. [1]).

The Zīj al-tashīlāt is not extant as such, but is probably the set of tables and accompanying explanation for a simplified method of computing planetary positions as worked out by Kāshī. In his Khāqānī Zīj ([23], ff.142r-155r) these tables occupy a section distinct from the planetary tables of standard type.

In addition to those listed in the Miftāh al-hisāb, the following treatises were written by or attributed to him:

Miftāh al-asbāb fī ʿilm al-zīj, listed in the Mosul catalogue [40].

Risālah dar sākht-i asturlāb listed in the Meshed catalogue

([38], Ms. math. 84).

Risālah fī maʿrifat samt al-qiblah min dāʾirat hindiyah, also listed in the Meshed catalogue.

Risālat ʿamal al-darb biʾl-takht wʾal-turāb is also included in the Tehran lithographed edition of 1889 of which a copy is in the Parliament Library in Tehran.

Al-risālat al-iqlīlāminah is mentioned by Kāshī himself in the Samarqand version of the Nuzhah ([19], p.311).

Kāshī's statements about the length of time required to complete the observations proved all too true. In the prolegomena to the zīj ([54], p.5) based on them, his royal patron Ulugh Beg, laments his death early in the course of the work. His collaborator and rival, Qādīzādah, also died before the zīj was finished. On the title page, the India Office copy of the Khāqānī Zīj [23] bears a note saying that on the morning of Wednesday, 19 Ramadān, 832 (22 June, 1429), at the observatory outside Samarqand there died the "mighty master, Ghiyāth al-Millah wʾal-Dīn, Jamshīd." According to Tabātabāʾī ([52], p.19) the incomplete copy of the same zīj in the Shrine Library at Meshed has the same annotation. That Kāshī left behind him more than his scientific works is witnessed by the British Museum manuscript of his Miftāh ([6], p.199) which has the following curious colophon:

> Verily I finished copying this honorable manuscript on the second day of the month of Shawwāl, year 997, (14 August, 1589). It was copied (or checked ?) by the one (who is) indigent (for the sake) of God, al-Razzāq, son of ʿAbdallāh, son of ʿAbd al-Razzāq, son of Jamshīd, son of Masʿūd, son of Jamshīd, the author of this noble book.

We lack evidence on which to pass judgment, or even to assess, Kāshī's personal character. Concerning him the author of the

Haft iqlīm (see [45], vol.ii, p.45) has this to say:

> The former (Kāshī) was ignorant of the etiquette of courts, but Ulugh Beg was obliged to put up with his boorish manners because he could not dispense with his assistance.

In the letter to his father he does not depict himself as the proverbially shrinking violet. At the same time this was a personal communication addressed to a parent, and presumably not intended for publication. And on the basis of the evidence at hand we can only agree with his own estimate - he was the best of the lot at Samarqand.

In the closing appendix to the revised Nuzhah, "On the Naming of the Instrument," ([19], p.312) Kāshī whimsically relates a suggestion from some of his friends, that his equatorium be called Jām-i Jamshīd,

> ... Jamshyd's Sev'n-ring'd Cup, where no one knows,

a likening of the instrument with its seven planetary deferents to the magic divining goblet of the mythical Iranian king, discoverer of the uses of wine, and whose name Kāshī bore.

> They say the Lion and the Lizard keep
> The courts where Jamshyd gloried and drank deep.

As for his scientific attainments and his place in the history of science, here we can operate from much firmer ground. He was first and foremost a master computer of extraordinary ability, witness his facile use of pure sexagesimals, his invention of decimal fractions (cf. [44] and [37], p.102), his wide application of iterative algorisms, and his sure touch in so laying out a computation that he controlled the maximum error and maintained a running check at all stages.

His equatorium marked the most extensive development ever given to this class of instrument. In particular his was the only mechanical device with which a determination of the planetary

latitudes was possible. If we retain mental reservations as to the practicality of the results, there can be no doubt of the ingenuity of the descriptive geometry involved.

He seems to have been a completely competent observer and astronomical technician, neither ahead of nor behind his times. The same statement can be made of his work in planetary theory. He accepted unreservedly the notion, not contained in the Almagest, that the moon, inner planets, sun, and other planets move in contiguous bands about the fixed earth, and that hence it was possible to compute in terrestrial units the mean distance of, say, Saturn from the earth. His contemporaries were therefore overgenerous in calling him "the second Ptolemy" ([33], vol.iv, p.21), but the next generation was equally sanguine in calling a mathematician of their own time "the second Ghiyāth al-Dīn Jamshīd" ([53], p.60).

The History of the Text and Its Versions

The Persian manuscript published in this volume is an apparently unique copy of a tract composed by some individual now unknown, between the years 1481 and 1512, and probably in Constantinople. This time spans the reign of the Ottoman Sultan Bayazid II, to whom the book is dedicated. Constantinople is mentioned in the text as this ruler's capital, and its longitude is taken as base for the planetary mean motion tables.

If the author is unknown, his prime source is not. He specifically states that the astronomical instruments he describes were invented by Jamshīd (al-Kāshī), and his work can be characterized as largely a translation into Persian of selected parts from Kāshī's own Arabic description of the same two instruments.

The latter book is called Nuzhat al-Hadā'iq (A Fruit-

Garden Stroll) and is extant in two versions. The first was written in Kāshān and completed on 10 February, 1416. The only copy known to have survived is Number 210 in [48], a microfilm of which has been made available by the officials of the India Office. The copy is modern, having been completed on 2 December, 1863. It is written in an easily legible nasta'līq hand, but very carelessly, and all the figures are missing. This Kāshān version we refer to as NK in the sequel.

In June, 1426, just three years before he died, Kāshī completed a rescension of the Nuzhah. This was after he had moved to Samarqand to work at Ulugh Beg's observatory there. Following the colophon of the original material, which was changed only in minor details, he added a set of ten appendices. These describe additional techniques for utilizing the instruments, and improvements or changes in the construction of their parts. It will be shown in the commentary (Section 42) to our text that the author made use of the Samarqand rescension, which we will abbreviate as NS. No manuscript copy is known to exist, but in 1889 it was printed in the Tehran lithograph edition of several of Kāshī's works. The example in the translator's possession is bound and paginated (pp.250-313) with the Miftāḥ al-Ḥisāb. It is written in a fair naskh hand, very carefully, and with text corrections in the margin. The space on page 261 for the main figure, however, has been left blank.

A table of contents of NS and NK follows, combined with a concordance of corresponding sections in our text.

Table of Contents of the Nuzhat al-Hadā'iq With a Concordance of the Kāshān (NK), Samarqand (NS, Tehrān lithograph), and Persian Versions

	NK	NS	Persian Anonymous	
	page	page	treatise and chapter	folio
Chapter I. ON THE CONSTRUCTION OF THE INSTRUMENT	3	250	I	5v
Construction of the Plate and Ring	4	250	I,1	5v
Laying Out the Deferents, etc.	4	251	I,2	7r
Laying Out the Latitude Marks, Sectors, etc.	8	252	I,3	8v
Construction of the Alidade and Ruler	10	253	I,4	11v
The "Tables" on the Alidade	13	254	-	-
The Difference Marks	15	255	I,4	12r
The Eclipse Marks	19	256	I,4	12v
On Laying Out the Mean Motion Table	22	258	I,5	13r
Table of Mean Motions	-	259	I,5	14v
Another Type, requiring no rotation of the plate inside the ring	24	260	-	-
Another Type, with movable ring, as before, the "parallel deferent type"	27	261	-	-
Another Type, also with movable ring, the "united deferent type"	30	263	-	-
Another, the "united apogee type"	33	263	-	-
Enumeration of type combinations	37	265	-	-
Chapter II. ON THE OPERATION OF THE INSTRUMENT	42	267	II	18r
Section 1. On the Extraction of Mean Positions	42	267	II,2	18r

		NK	NS	Persian Anonymous	
		page	page	treatise and chapter	folio
Numerical Example		43	268	-	-
Section 2.	On the Arrangement of the Instrument	46	269	II,1	18r
Section 3.	On the Determination of the Sun's True Longitude, Equation and Distance	47	270	II,3	19v
Section 4.	On the Determination of the True Longitudes of the Other Planets	48	270	II,4 II,5	20r 21r
Section 5.	On the Determination of Equations	57	274	II,6	21v
Section 6.	On the Determination of the Distances of Planets from the Earth	58	274	II,8	26v
Section 7.	On the Determination of Retrogradations and Forward Motions	60	275	II,9	28r
Section 8.	On the Knowledge of (the Planetary) Sectors	64	276	II,10	30r
Section 9.	On the Determination of Planetary Latitudes	66	277	-	-
Section 10.	On the Determination of the Lunar Third Equation (i.e., the correction due to the moon's orbit being inclined with respect to the ecliptic) and its (the moon's) True Longitude in the Parecliptic	72	280	-	-
Section 11.	On the Determination of Lunar Eclipses	73	280	II,11	31r
Section 12.	On the Determination of Solar Eclipses	76	281	II,12	33r
Section 13.	On the Determination of the Mean of Transfer by Means of the Sun's True Longitude	77	282	II,13	34v

	NK	NS	Persian Anonymous	
	page	page	treatise and chapter	folio
Section 14. On the Determination of True Altitude from the Apparent, and Vice Versa, and Parallax in the Altitude Circle	78	283	II,14	35r
Section 15. On the Determination of the Excess of Revolution by Means of the Altitude, and Mention of the Rest of What Can be Determined with this Instrument	80	283	-	-
Conclusion. ON THE MAKING OF THE "PLATE OF CONJUNCTIONS", AND AN EXPLANATION OF THE OPERATION(S PERFORMED) WITH IT	83	285	I,Concl. 15r II,Concl. 37r	
Colophon	91	288	-	-
Appendix 1. On Using an Oval Deferent for the Moon	-	289	-	-
Appendix 2. On Laying Out Oval Lunar and Mercury Deferents	-	289	-	-
Appendix 3. On the Planetary Latitudes	-	291	I,3 II,7	22r
Appendix 4. On How to Fix the One Ruler on the End of the Other	-	295	-	-
Illustration of Parallel Rulers	-	298	-	-
Appendix 5. On the Determination of Mean Positions without Computation	-	298	-	-
Appendix 6. On an Improved Type, Better than those Already Described	-	306	-	-
Appendix 7. Directions in Case the Plate is Large	-	307	-	-
Appendix 8. On the Determination of the Equation of Time	-	311	II,15	36r

	NK	NS	Persian Anonymous	
	page	page	treatise and chapter	folio
Appendix 9. On the Explanation of the Operation as Mentioned in Appendices 6 and 7	-	311	-	-
Appendix 10. Relating the Naming of the Instrument	-	312	-	-
Colophon of the Supplement	-	313	-	-

TEXT
and
TRANSLATION

The numbers along the left edge of each page of the translation identify corresponding lines of the text facsimile on the opposite page. Restorations to the text are indicated by the usual square brackets; in general, words enclosed in parentheses have no equivalents in the text, but have been added to clarify the meaning of the relevant passage.

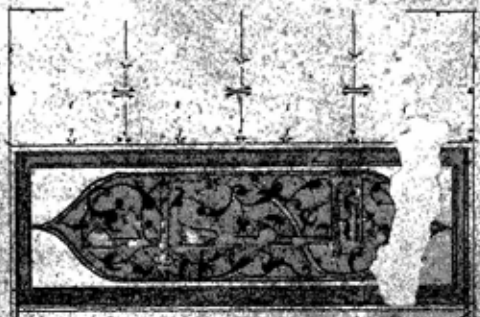
Marginal additions to the text are of two kinds, scribal omissions from our copy, and explanatory notes. The former were written in, perhaps at the time the copy was made, presumably by checking it against another copy. These have been inserted into the body of the translation, set off by asterisks at the beginning and end of the insertion. The marginal notes proper, however, have been translated as such and appear in the margins of the translation.

رسالة في العمل باسهل آلة من قبل النجوم

f.2r

A Short Work on the Operation of the Easiest Instrument Having to Do with the Planets*

*This title, in Arabic and written in a hand different from that of the rest of the manuscript, need not be regarded as the original title of the work.



شکر و سپاس بیقیاس و ثنای بی‌قیاس لایق حضرت
عزت است تعالی شانه که اطباق سموات و ابدار
غبرا بی‌عمد و ثوابت و جواهر زواهر
سیارات ثوابت مرصع گردانید حکمی که
مقتضای کاملهٔ اش حدوث مکونات سفلی را
مرتبط باوضاع علوی ساخت قادری که بدست
قدرت و تقدیر چندین اشخاص مختلف را در
قالب تصویر آورد تعالی الله الخالق البارئ المصور
له الاسماء الحسنی و صلوات و
تحف تحیات بر قطب فلک رسالت و مدار

f.2v

TO THE PRAISE OF GOD

Thanks and [praise][1] and incomparable glory is the due of the Power,[the Creator][2] who has inlaid the layers of the heavens with glistening pearls, with shooting [stars][3], and glittering jewels, the planets and the fixed (stars). The Sage who, impelled by His complete [wisdom][4], wrought the bringing into existence of the inferior (planets) together with the situations of the superior (planets); the (Al)mighty, who by the hand of power and destiny moulded how many shining bodies. "Verily his are the creation and the command; blessed be God, the best of Creators"[5], and benediction and praises and the gift of salutation upon the Pole of the Heaven of Prophecy and the Circlet of the

---

1. Wormeaten, read سپاس
2. " , " صانعی
3. " , " بکواکب
4. " , " حکمت
5. Qur'ān, 7:52 (ed. of Flugel).

آسمان جلالت محمد مصطفی و بر آل و اصحاب او
که سعود [illegible] و بعد
پوشیده نماند که شریفترین نوعی از انواع علوم ریاضی
علم نجوم است که نفس انسانی را بدو اقتدا آن شرف
اطلاع بر کیفیات حرکات کواکب در طول و عرض
و کیفیات اوضاع ایشان و مقامات هریک
در بروج و نطاقات و معرفت اوقات صلوة
و سمت قبله و سایر بلاد و ابعاد و اجرام هریک
از کواکب از زمین و مساحت میان بلاد حاصل
میشود و حصول جمله این مقاصد بسیار نه مگر از درج
و معرفت آن موقوفست بحساب از تفریق و
تضعیف و جمع و تصغیف و ضرب و قسمت و
اعمال بعضی از تذییلات و صنعوت آن ظاهر

f.3r

Sky of Glory, Muhammad the Chosen (One) and upon his descendants and his companions,
who are the beneficent (planets) in the sky of leadership and the sphere of the stars of imitation. However,
let it not remain covered that the most honorable branch of the branches of the science of mathematics
is astronomy, which supplies the human soul with the acquiring of
that honor (i.e. astronomy), knowledge of the amount of the planetary motions in longitude and latitude,
and the modes of their conditions, and the stations of each one
in the zodiacal signs, and the (planetary) sectors, and the knowledge of the times of prayer,
and the direction of Mecca, and the other cities, and the distances of the bodies of each
of the planets from the earth, and the distance between cities.
But the attaining of all these objectives is impossible except by (means of) an astronomical handbook (a zīj),
and the knowledge of it is dependent upon computations: on subtraction, and
halving, and addition, and doubling, and multiplication, and division, and
operations with various (astronomical) equations, and its difficulty is apparent

وبیدا وروشن میدانست واستاد محقق ومحرر
مدقق مکمل علوم اوایل وکاشف معضلات مسائل
افتخار الحکماء فی العالم مولانا الاعظم مولانا غیاث
الملة والدین جمشید بن مسعود طاب ثراه مصحف آلتی تصور کرده
واسم اطبق المناطق نام نهاده ودر رساله در کیفیت
صنعت وعمل آن پرداخته که بدان آلة معمولات
زیج باسهل طرق واقل زمان عمل کرده می شود
وچندان احتیاج بضرب وقسمت حساب
ندارد والی زماننا هذا سیج احد از آحاد علما
باوجود آنکه همگی همت بر اجتماع مآثر فضل و
افضال مقصور داشته اند نقاب حجاب از
چهره آن مخدره برداشته اند واز پس پرده
بیرون عالم رو زیبا ننموده واین تقصیر موقوف

f.3v

and known and clear and manifest, and so the meticulous master, the investigator, the scrutinizing author, the completer of the primary sciences and the solver of difficult questions, the pride of the savants of the world, our most mighty master, our master Ghiyāth al-Millah w'al-Dīn (the Refuge of the Congregation and of the Faith) Jamshīd, may God cool his resting-place, invented an instrument and called it the Plate of Heavens, and prepared a treatise describing its construction and operation, such that with this instrument the problems commonly solved with a zīj may be solved by the easiest methods and in the shortest time, and it does not require much multiplication and division and computation. And up to this our time, no single one of the scholars, notwithstanding all the effort toward the carrying forward of learned and excellent illustrious deeds, they were deficient. The veil-curtain from before the face of that virgin maiden they did not raise, and from behind the curtain of existence to the world she was not made manifest. But this destitute one, by grace of

ارادت بیچون و یباین دولت روز افزون
پادشاه جهان و فرمان فرمای دوران سلطان
آسمان سریر خورشید ضمیر ستاره لشکر مشتری منظر
ظل الله فی الارضین و قهرمان الملک والطین
قامع الکفرة والمشرکین بیت

آیت فایدة حق صورت امن و امان

نص کتاب ظفر مهد کی آخر زمان
السلطان بن السلطان ابوالفتح سلطان بایزید
بن سلطان محمد خان خلد الله ملکه وسلطانه و ابد علی
علی العالمین برّه و احسانه این آیت را ساخت
و تعویم کواکب و عروض ایشان و خسوف
و کسوف عمل کرد و بازیج موافق کرد ایشد
و در آنچه عمل کردیشد بود عجالة الوقت را و در

f.4r

the ineffable will and bounty of the daily-increasing government of the Ruler of the World, the Issuer of Commands of the Ages, the Sultān of the Heavenly Throne, the Sun of Conscience, the Star of the Jupiter-appearing Army, the Shadow of God on both Worlds, the Champion on Sea and Land, the Subduer of the Infidels and Idolaters. A Couplet:

> The symbol of (what is) supported by Right (i.e., God), the aspect of peace and security,
> (He is) the very letter of the book of victory, the Mahdī and the Last of (this) Age,

the Sultān, son of Sultān Abū al-Fath, Sultān Bāyazīd, son of Sultān Muhammad Khān, may God perpetuate his dominion and his kingdom, and may his beneficence and his goodness endure upon both worlds, (the author) constructed this instrument, and operated (with it to find) the true longitudes of the planets, and their latitudes, and lunar and solar eclipses. (The results) agreed with (those obtained by use of) a zīj. And of whatever has been done at present (i.e., this work itself) having been

چند نوشته معروض مسئله سنیه و عجبه علیه
گشته فان وقع فی حیز القبول فهو غایة
المامول و نهایة المسئول ملتمس انکه چون بر سنوی
و خطکشیده اصلاح یابند بقلم سبکاره و خامه کوتاه
نگارد اصلاح فرماینده و ترتیب این رساله بر دو
مقاله لایق افتاد مقاله اول
در صنعت طبق المناطق مشتمل بر پنج باب و خاتمه
باب اول در صنعت قرص و حلقه
باب دوم در رسم ادجات دوایر
و مناطق و نقطه محاذات و معدل للسیر باب
سیوم در رسم قطر استوا و نقطه عرض و خطوط
آن باب چهارم در صنعت عضاده
و بسیط و تقسیم آن باب پنجم در رسم

f.4v

written on a few sheets, it is presented to the mighty and glorious (one), the Sublime Porte.

And if acceptance be possible it (would be) the extreme of

that which is hoped for, the utmost of that which is requested. It is requested that if information of a slip

or a mistake is found (it) be corrected with the musk-dripping pen and the reed (pen which is) the pearl-portrayer.

And the arrangement of this disquisition came to be worthy of two

treatises. (Contents of) THE FIRST TREATISE:

On the Construction of the Plate of Heavens, containing five chapters and a conclusion.

Chapter One, On the Construction of the Disk and the Ring.

Chapter Two, On the Drawing of the Apogees, and the Centers,

and the Deferents, and the Opposite Point, and the Equant. Chapter

Three, On the Drawing of the Equating Diameter, and the Latitude-Point, and their Lines.

جداول اوساط و غیرها خاتمه در
صفت لوح اتصالات مقاله دوم
در عمل باسن آلت مشتمل بر بانزده باب
اول در ترتیب آلت باب دوم در استخراج
اوساط کواکب باب سیوم در استخراج
تقویم آفتاب باب چهارم در استخراج
تقویم قمر باب پنجم در استخراج
تقویم کواکب متحیره باب ششم در
معرفت بعد ملات باب هفتم در
معرفت عروض کواکب باب هشتم در
معرفت ابعاد کواکب از مرکز عالم باب نهم
در معرفت رجعت و استقامت باب دهم
در معرفت مطالعات اوجی و تدویری باب

f.5r

The Table of Mean (Motions), and so on. Conclusion, On
the Construction of the Plate of Conjunctions.
(Contents of) THE SECOND TREATISE:
On the Operation of this Instrument, contained in fifteen chapters: Chapter
One, On the Arrangement of the Instrument. Chapter Two, On the Extraction
of the Mean (Motions) of the Planets. Chapter Three, On the Extraction of
the True Longitude of the Sun. Chapter Four, On the Extraction of
the True Longitude of the Moon. Chapter Five, On the Extraction of
the True Longitudes of the Planets. Chapter Six, On
the Determination of Equations. Chapter Seven, On
the Determination of Planetary Latitudes. Chapter Eight, On
the Determination of the Distances of the Planets from the Center of the Universe. Chapter Nine, On
the Determination of Retrogradations and Stations. Chapter Ten
On the Determination of Apogee and Epicyclic Sectors. Chapter

یازدهم در معرفت خسوف باب دوازدهم
در معرفت کسوف باب سیزدهم در معرفت
وسط تحویل باب چهاردهم در معرفت
ارتفاعی حقیقی ادمنی و معرفت ارتفاع مرئی
از حقیقی و اختلاف منظر باز ارتفاع باب
پانزدهم در معرفت تعدیل ایام بلیالی خاتمه
در عمل بلوح اتصالات مقاله اول
در صنعت آلت موسوم بطق المناطق مشتمل بر پنج
باب و خاتمه باب اول در صنعت قرص
و حلقه چون خواهند که این آلت را بسازند قرصی
صحیح الاستدار بسازند از نحاس یا صفر
یا شبه یا خشب صلب همچو صفحه اسطرلاب
هر چند بزرگتر باشد بهتر بود اما قطر او یک

f.5v

Eleven, On the Determination of Lunar Eclipses. Chapter Twelve,
On the Determination of Solar Eclipses. Chapter Thirteen, On the Determination of
the Mean of Transfer. Chapter Fourteen, On the Determination of the
True [from]* the Apparent Altitude, and the Determination of the Apparent Altitude
from the True, and Parallax in the Altitude Circle. Chapter
Fifteen, On the Determination of the Equation of Time. Conclusion,
On the Operation of the Plate of Conjunctions.

THE FIRST TREATISE

On the Construction of the Instrument Called the Plate of Heavens, containing five
chapters and a conclusion. CHAPTER ONE. On the Construction of the Disk
and the Ring. If it is desired to make this instrument, make a disk,
truly circular, of copper, or brass,
or yellow copper, or hard wood, like the plate of an astrolabe,
the bigger the better. Its diameter should be at least one

*For او read از.

ذراع بود و اگر دو پاره ذراع بود بهتر باشد چه
عمل صحیح می آید و بر دو حلقه ترکیب کنند چون پرگار
اصطرلاب چنانکه صفحه در وی حرکت کند و از وی نشکند
نگردد و سطح حلقه با سطح صفحه چون یک سطح مستوی
گردد بدان طریقه که آورده محیط صفحه زبانه باریک
بیرون آرند و در تمام معدل حلقه حفری کنند تا آن زبانه
در وی در آید و استوار آید و را بسطره و شاقول امتحان
کنند چنانکه بمعلوم صناع است پس بر مرکز قرص
سه دائره بر حلقه رسم کنند و دایره اول را بدو اره
قسمت راست کنند تا دو آیر پنجگانه بدو منقسم
شود و در مابین اول و ثانی اسامی بروج بنویسند
و توالی از جانب یسار اعتبار کنند چون خود را
بر مرکز قرص فرض کرده باشند و ثانی را بهنیات

f.6r

cubit, and if it be two or three cubits, operating with it will be more accurate. And mount around it a ring, like the raised ring (around the plate) of an astrolabe, in such fashion that the plate move in it and not become separated, and the surface of the ring and the surface of the plate should form a single plane surface, and in such fashion that from the circumference of the plate a narrow tongue be extended. And in all the concavity of the ring let a hollow be cut out so that the tongue fit into it. The flatness of this should be tested with ruler and plumbline, as is known to the craftsman. Then, around the center of the disk, draw five circles on the ring, and divide the first circle into twelve parts, so that the fivefold circles are divided by them, and between the first and the second write the names of the (zodiacal) signs, and consider the succession from the left, since it has itself been assumed around the center of the disk. The second is divided into seventy-

ود و قسم کرده عدد درجات را بحکانه در مابین ثانی
و ثالث رقم نهند و ثالث را بسیصد و شصت بعد و
درجات فلک البروج قسمت کنند و راج را بکسور بر
درجات بقدر امکان قسمت کنند و خطوط وصل
کنند میان رابع و خامس بجهت کسور و خطوط بهر
قسمی از اقسام حاوی تا دائر که اندرون سمت است
کنند و اگر خطوط اقسام خمسات بلدی دیگر کنند
بجهت تسهیل اعمال بهتر بود پس بر دائر خامس که
بکسور منقسم است بر محاذات هر قسمی ثقبی دقیق
کنند چنانکه مثقبه از حلقه بگذرد و زبانه قرص نیز
بر محاذات او سوراخ شود و سه اثقاب باید که بر
محیط یک دائر بود بسازند که بهر یک از ان
اثقاب که فرو برند حلقه بر قرص ثابت شود

پس عمل باریک

f.6v

two parts, the number of five(-degree intervals around the circle). Between the second and the third place numerals. And the third (circle), divide it into three hundred and sixty, the number of the degrees of the zodiac, and the fourth into fractions of degrees to the amount possible, and join lines between the fourth and the fifth for the fractions, and the lines of each of the inner parts, extend (them) to the innermost circle. If the lines of the five parts be made of different color(s) for ease of operation it will be better. Then on the fifth circle, which is divided into fractions, opposite each division, have a careful hole so that the drill pass through the ring. And the tongue of the disk also should be pierced opposite it, and all the holes must be along the circumference of one circle. *Then* make *a very thin* peg so that whichever of those holes it may pass through, the ring will be fixed on the disk,

---

* Marginal correction.

واین حلقه قایم مقام فلک البروج بود باب
دوم در رسم اوجات و مراکز و مناطق و نقطه
محاذاة و معدل المسیر بر محیط صفحه کیف ما اتفق
نقطه فرض کنند و انرا اوج شمس نامند پس ازین
نقطه بمقدار مابین اوج شمس و اوج هر کوکبی
بر توالی البروج احذ کرده علامات بگذارند

مابین اوج آفتاب و کواکب خمسه				
زحل	مریخ	زهره	عطارد	قمر
۷۰ کج	۱۵ ط ۱۰	۱۰ و ۵	۱۵ ط ۱۰	۲۵ ح ۴

پس میان هر علامت و مرکز بخطی مستقیم وصل کنند
چنانکه بعد اتمام آلت محو آن ممکن باشد و از مرکز
صفحه بسوی اوج هر کوکبی غیر از شمس و برای شمس
بسوی حضیض او و برای قمر بسوی عبدالله اجزاء

f.7r

and this ring shall stand in the place of the zodiac. CHAPTER TWO. On the Drawing of the Apogees, and Centers, and Deferents, and the Opposite Point, and the Equant. On the circumference of the plate, however it may befall, assume a point, and call it the sun's apogee. Then, from this point, to the amount between the sun's apogee and the apogee of each planet, having taken (it) in (the direction of the) succession of the signs, place marks.

(Angles) Between the Apogee(s) of the Sun and the Five Planets				
♄	♃	♂	♀	☿
$5^{s}10;28^{o}$	$2^{s}29;16^{o}$	$1^{s}16;5^{o}$	$11^{s}19;15^{o}$	$4^{s}2;40^{o}$

Then, between each mark and the center, join a straight line such that, after the completion of the instrument, erasure be possible. And from the center of the plate, in the direction of the apogee of each planet except for the sun, and for the sun in the direction of its perigee, and for the moon in the direction of the beginning of divisions,

بمقدار ما بین المرکزین بگیرند و بر آن علامتی نهند که
اثر آن باقی ماند و آن علامت مرکز حامل آن
کوکب بود بدین تفصیل که مذکور می شود و این

ابعاد مراکز حوامل کواکب علی ان نصف قطر الصحیح ستون

ب ز ط	ب ط	ب غ	ت لب	ع ح	ات	ع نب

بمقدار بیست که نصف قطر صفحه شصت درجه باشد
پس هر یکی از این علامات را مرکز ساخته بر آی هر یک
از قمر و زحل و مشتری و مریخ و زهره و عطارد این
بعد و این رسم کنند و این دو اید مناطق حوامل این

انصاف اقطار حوامل علی ان نصف قطر الصحیح ستون

ط ما	نب ب	نه لج	نه ک	غ غ

f.7v

to the amount between the two centers, lay (it) off, and place a mark on it that its trace shall remain. And that mark shall be the deferent center of that planet, according to the particulars recorded herewith. And these

Distances of the deferent centers of the planets (in units) such that half the plate diameter is sixty.						
☉	☾	♄	♃	♂	♀	☿
2;6,9	10;19	2;58	2;32	4;33	1;2	4;52

(units) are of such an amount that half the plate diameter shall be sixty degrees. Then, each one of these marks having been made a center, for each one of the moon, and Saturn, and Jupiter, and Mars, and Venus, with this distance (i.e., radius) draw a circle, and these circles are the deferent heavens of these

Halves of the deferent diameters (in units) such that half the plate diameter is sixty.				
☾	♄	♃	♂	♀
49;41	52;2	55;28	45;27	58;58

کواکب است و احتیاج برسم منطقه خارج شمس
نیست چه محیط صفحه را منطقه شمس فرض کنند مستعار
و مرکز مرسوم را مرکز مستعار شمس نامند و آن
مرکزی که برای عطارد رسم کرده اند مرکز مدیر عطارد
باشد پس از مرکز مدیر خطی خراج کنند مقاطع
خط ابعدی بر قابیه و از آن مرکز بهر دو جانب تمکن
و یسار خط اوجی بمقدار ۵ ح دو علامت بگذارند
و بهر یک ازین علامتین را مرکز ساخته بگذارند
تا کج دو قوس رسم کنند تا شکلی اهلیلجی حاصل
شود که نصف قطر اطول او ناح و نصف قطر
اقصرش مو به بود و این بعد از مرکز تدویر عطارد
بود و ما آنرا منطقه عطارد نامیم و باید که بهر یک
از منطقه کوکبی را بلونی دیگر کنند تا در وقت عمل

f.8r

planets, and there is no need for drawing the eccentric heaven of the sun,
since the circumference of the plate is ficticiously assumed (to be) the heaven of the sun,
and the marked center (of the plate) is called the ficticious (deferent) center of the sun. And that
center which has been drawn for Mercury, let it be Mercury's turning center.
Then, from the turning center extend a line intersecting
the line of apsides perpendicularly, and from this center to both sides, right
and left of the line of apsides, to an amount 5;8, make two marks,
and, having made each one of these two marks a center, with distance (i.e., radius)
51;23 draw two arcs so that an elliptical-shaped figure results,
of which half its larger diameter is 51;8 and half its
shorter diameter is 46;15, and this is the orbit of Mercury's epicycle center,
and we call (it) Mercury's deferent. And each
of the planet's deferents must be drawn in a different color, so that at the time of operation

شبیه کرده و پس از مرکز منطقه هر یک از علویه و زهره
جدا کرده و در قسم از مرکز صفحه بجانب مخالف
ببداء اجزا یعنی بطرف میزان بقدر بعد مرکز آن
کوکب از مرکز صفحه نگیرند و بران علامتی نهند
که امتحان باقی ماند و در عطارد بر منتصف مابین مرکز
صفحه و مرکز تدیر علامت کنند و این علامات را
در غیر قمر مرکز معدل المسیر و در قمر نقطه محاذات خوانند
باب سیوم در رسم قطر استوا و نقطه عرضی
و خطوط عرض و مبادی خطاقات و صورت صفحه
و مناطق و علامات و دوایر و خطوط بر صفحه قطری
رسم کنیم که مبداء اجزا و محیط گذرد و آنرا قطر
استوا نامیم پس بر قطر استوا بقرب اول
میزان هشت علامت کنیم چنانکه اثر علامات

f.8v

no mistake be made. Then, from the deferent center of each of the superior (planets), and Venus
on the side of its apogee, and for the moon from the center of the plate on the opposite side from
the beginning of divisions, i.e. on the side of Libra, to the amount of the distance of the center of that
planet from the plate center, lay (this distance) off, and put a mark on it
that its trace remain. And for Mercury, on the half(-distance) between the center of
the plate and the turning center make a mark, and these marks,
except for the moon, are the equant centers, and for the moon it is called the opposite point.
CHAPTER THREE. On drawing the Equating Diameter, and the Latitude Point(s),
and the Latitude Lines, and the Beginning of the Sectors, and the Picture of the Plate,
and the Deferents, and Marks, and Circles, and Lines. On the plate draw a diameter
which passes through the beginning of the divisions and the circumference, and we call it the equating diameter.
Then, on the equating diameter, near the first (point) of
Libra, we make eight marks such that the traces of the marks

باقی ماند دو برای زحل دو دو برای مشتری و
دو برای مریخ یکی برای زهره و یکی برای عطارد
و بعد هر علامتی از مرکز صفحه بدین تفصیل است

ابعاد وسط عرض الکواکب الخمسة علی ان نصف قطر الصفحة									
زحل		مشتری		مریخ		زهره		عطارد	
خ ن ه	۵ ا	ن ه	خ ط	۵ ۱	م ند	خ ح	ن ه	ح ه	ه ۱

و این علامات را نقطهٔ عرض خوانیم و اگر برای
هر یک از علویه نیز بعلامات واحده اکتفا کنند
در منتصف بعدین در مقصود خلل نه بد پس
بر مرکز صفحه نصف دائره رسم کنیم بر احد جانبین قطر
استوا اگر بر جانب بروج جنوبی بود بهتر که
نصف قطر او مساوی حصه نه درجه باشد و منقسم
باجزاء حجره بعد ازان سطاح را بر هر دو

f.9r

remain, two for Saturn, and two for Jupiter, and two for Mars, one for Venus, and one for Mercury. And the distance of each mark from the center of the plate is detailed herewith:

Distances of the latitude line(s) of the five planets (in units) such that half the plate diameter is sixty							
♄		♃		♂		♀	☿
53;55	50;1	57;46	53;9	50;0	40;54	58;58	46;0

and these marks we call latitude point(s), and if, for each of the superior (planets) also one is content with halving (the sum of) the two distances the objective will not be marred. Then, with the center of the plate (as center) draw a semicircle on one side of the equating diameter, if on the side of the southern signs (it is) better, such that half its diameter be equal to the sine of nine degrees of the concavity of the ring. After that, place a ruler upon pairs of

In the case of the inferio (planets) the mark(s) have been made single, because they do not have two extremes, for the epicycle center of Venus is always north and of Mercury alway south.

This is the distance of the planet's epicycle center from the center of the universe in northern and southern directions.

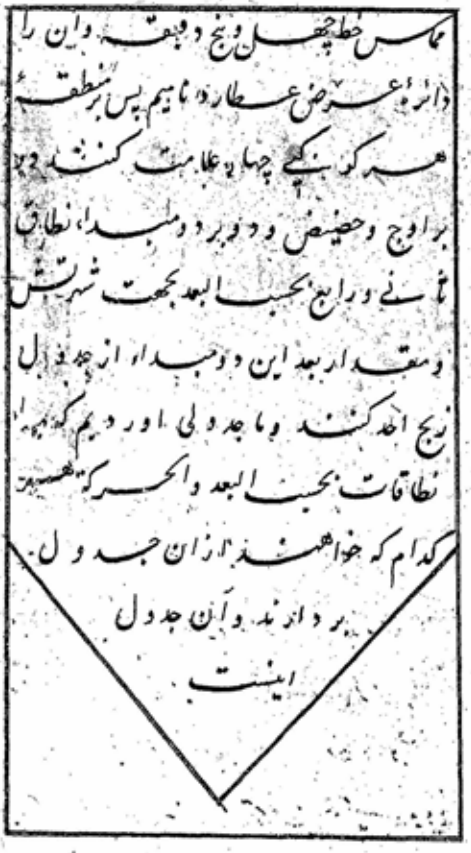
تا نصف دایره

جزء متساوی بعد از قطر استوا، بهند
و خطی می کشند در داخل نصف دایره خطوط متوازیه
که دوری قطر استوا بود و متقاطع نشود و لا محاله ابعاد
بین خطوط بنسبه خوب قسمی خواهد بود از یکی
تا به پس بر محیط نصف دایره اعداد خطوط را متوالیه
از جانبین بنویسند و همچنین خطوط دقایق را نیز
تا ممکن رسم کنند و این خطوط را خطوط عرض نامیم
و همچنین بر مرکز صفحه نصف دایره رسم کنند
که مماس خطی درجه از خطوط عرض باشد این را
دایره عرض قمر خوانیم و اگر صفحه بزرگ باشد
رسم خطوط بدقیقه دقیقه یا دقیقتین کرده باشند
دو دایره دیگر رسم کنند یکی مماس خط ده دقیقه
دایره دایره عرض اول رسمش خواهیم و دیگری

f.9v

points equidistant from the equating diameter,
and draw line(s) inside the semicircle *(up) to the semicircle* in parallel lines
which are parallel to the equating diameter (and such that the inside) be filled. And undoubtedly the distances
between the lines will be in the ratio of the sines of the parts from one
to nine. Then on the circumference of the semi-circle, write the numbers of the successive lines
from both sides, and in the same fashion draw also the lines of the minutes
(to the amount) possible, and these lines we call latitude lines.
And in the same manner, draw a semicircle about the center of the plate
which shall be tangent to the five-degree line of the latitude lines.
We call this the moon's latitude circle. And if the plate is large
the lines may have been drawn minute-by-minute or (every) two minutes.
Draw two more circles, one tangent to the ten-minute line,
and we call it the circle of Venus' first latitude; and the other

*Marginal addition.


مماس خط چهل و پنج درجه و این را
دائرهٔ عرض عطارد نامیم پس بر منطقهٔ
هر کوکبی چنانکه علامت کند دو
بر اوج و حضیض و دو بر دو مبدا نطاق
ثانی و رابع بحسب البعد بجهت شهریش
و مقدار بعد این دو مبدا از چند اول
زیج اخذ کنند و ما جدولی اورده ایم که بهر
نطاقات بحسب البعد و الحر که تعیین
کدام که خواهند از ان جدول
بر دارند و آن جدول
اینست

f.10r

tangent to the forty-five minute line, and we call it*
the latitude circle of Mercury. Then, on the deferent of
each planet make four marks, two
on the apogee and perigee, and two on the two beginnings of the second and fourth sectors
reckoned according to distance, as is known (for each particular planet).
And the amount of the distance of these two beginnings
is taken from a table of the (Khāqānī) Zīj, and we have brought (out) a table such that
the sectors, computed according to distance and motion, any
which is wanted, take it from that table,
and that table
is this:

*The word آزا is repeated in the text.

جدول مبادی نطاقات

ارکان	نطاق الاوجی بالمرکز				نطاق التدویری فالمماسة المعدلة			
	اول	ثانی	ثالث	رابع	اول	ثانی	ثالث	رابع
	٤٢٤	ح ٤ ا	و ٤ ا	ح ك ط ا	٤٢٤			
		ب ك ب		ط ه خ		ح ب ك ا	و ٤ ا	ح ك ح
		ح ك ه		ح ك د		ح ب ه		ح ك ك
		ح ح ك		ح ك ك		ح ر ك		ح ك ك
		ح ح ك		ح ك ك		ى ح ح		ر ب ب
		ح ا ا		ح ك خ		ح ك ح		ح ط ب
		ب ر ه		ط ك ب		ح ط ح		ا ك ك
		ح ب ا		ح ك ا				
		ح ك ا		ح ك ا		ح ه ح		ح ك ب
		ح ح ب		ح ك ب		ح ه خ		ح ك ب
		ح ب ك		ح ك ك		ح ب ك		ح ط ك
		ح ب ه		ح ك ح		ى ر ح		ر ك ب
		ح ا ح		ح ك ا		ى ر ا		ر ه ا
		ى ح ب		ح ك ح		ح ط ا		ح ب ظ

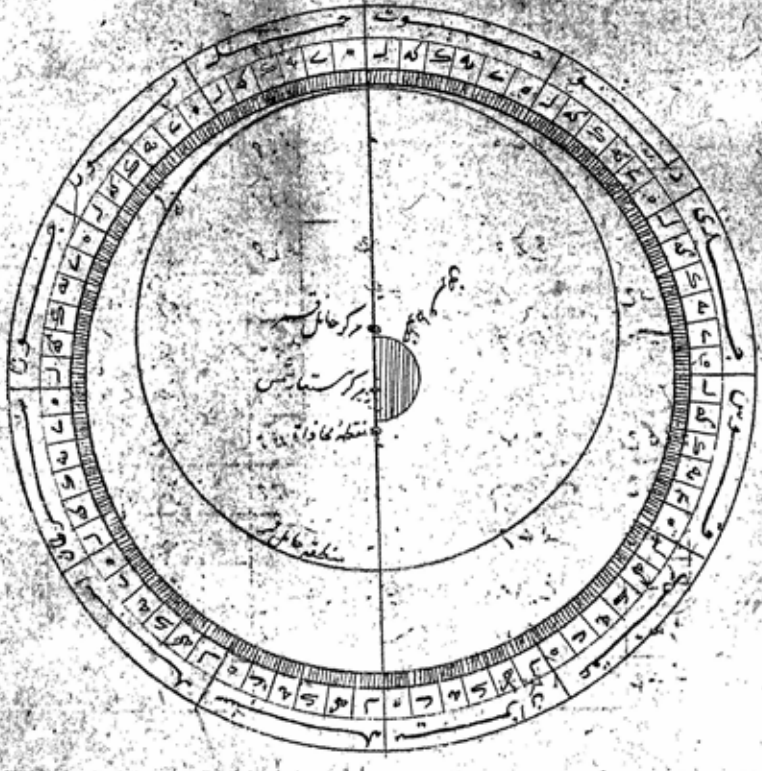
f.10v

TABLE OF THE BEGINNINGS OF THE SECTORS

	The Planets	Apogee Sectors, (according) to the Center				Epicycle Sectors (according) to the Adjusted Anomaly			
		First	Second	Third	Fourth	First	Second	Third	Fourth
According to Distance	☉	0°0;0°	3°1;0°	6°0;0°	8°29;0°	0°0;0°			
	☾		2 24;2		9 5;58		3°2;31°		[8]°27;2[9]°
	♄		3 4;54		8 25;6		3 2;56		8 27;4
	♃		3 3;26		8 26;34		3 [5];3[8]		8 2[4];22
	♂		3 8;30		8 21;30		4 17;48		7 12;12
	♀		3 1;1		8 29;59		3 20;43		8 9;[17]
	☿		2 7;44		9 22;16		3 9;[2]3		[8] 20;37
According to Motion	☉		3 2;1		8 28;[59]				
	☾		3 24;0		8 6;0		3 5;3		8 24;57
	♄		3 3;16		8 26;44		3 5;53		8 27;7
	♃		3 2;37		8 27;23		3 10;34		8 19;26
	♂		3 5;42		8 24;18		4 7;38		7 22;22
	♀		3 1;[0]		8 29;0		4 15;0		7 15;0
	☿		3 3;2		8 26;58		3 19;1		8 10;59

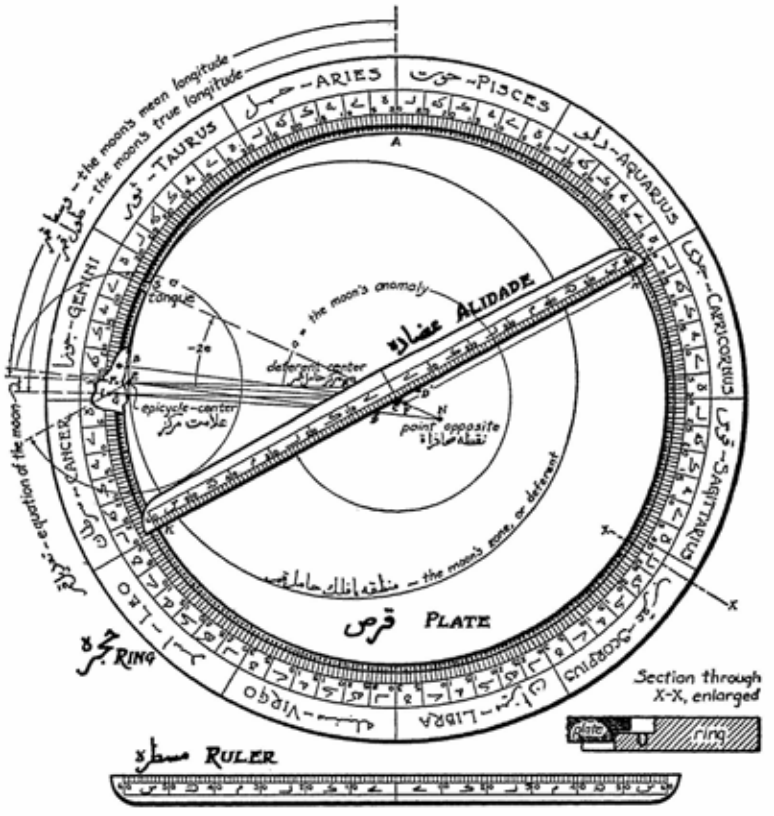
صورت مجموعه صفحه دساطیق وعلامات ودوایر

وخطوط اینست



f.11r

The Picture of the Ring and Plate and Deferents and Marks and Lines is this:



(Figure 1)

باب چهارم در صنعت عضاده و
مسطره و دو سطره بسازند از نحاس یا صفر
یا خشب یکی مثل عضاده اصطرلاب که او را
حرف واحد باشد و باید که طول او زیاده باشد
از قطر صفحه و اقصر از قطر محدب حجره
و حرف او را از مرکز تا مقدار نصف قطر
صفحه از جانبین بشصت قسم کنند مثل عضاده
اصطرلاب محک و اعداد اجزای او را از هر دو
جانب مرکز طرد و عکسا بنویسند و باید که
موضع مرکز و قطب هر و نصف دائره باشد
و در تقعیر آن زائده بقدر امکان گویند تا مرکز
بسیاری در ثبات او مختفی نماند و دیگری نیز
بهمان مقدار و اجزا لکن بجای زائده مسطره

f.11v

CHAPTER FOUR. On the Construction of the Alidade and Ruler. Make two rulers, of copper, or brass, or wood, one like the alidade of an astrolabe, such that (it) shall be one (straight-)edge, and its length must be in excess of the diameter of the plate and shorter than the convex diameter (i.e., outer diameter) of the ring. And its edge, from the center to the amount of half the diameter of the plate, from both sides divide it into sixty parts, like the alidade of a sine astrolabe, and the numbers of its divisions are written proceeding in opposite directions from the center, and the place where the pivot passes (through it) must be (on a projecting) semicircle, and in making that (semicircular) projection as small as possible, care should be exercised that the center not be hidden in its slope. And the other (ruler) is also (made) the same size, and with the same divisions, but in place of the projection on the

اوّل در دو جهتی مستدیر کند که راندهٔ اوّل
عند الحاجة در و در آید و هر دو مسطرهٔ چون
مسطرهٔ واحد کرده و اعداد مسطرهٔ ثانیه
را از یک طرف تا طرف دیگر بنویسد
طرد او عکسا و با مسطرهٔ اوّل را عضاد
و مسطرهٔ ثانیه را مسطرهٔ دایم پس
بر احد جانبین عضاد شش علامت کند
بجهت کواکب سته غیر شمس و ازار ارقام
اختلاف نامیم و ابعاد علامات از مرکز صفحه
بنحوی مر کوکبی بدین منفصل است

ابعاد ارقام الاختلاف علی ان نصف قطر الصفحة کون

قمر	زحل	مریخ	مشتری	زهره	عطارد
٥ لز	٠ لح	٧ لح	لـ لـ	مـ کله	لح لـ

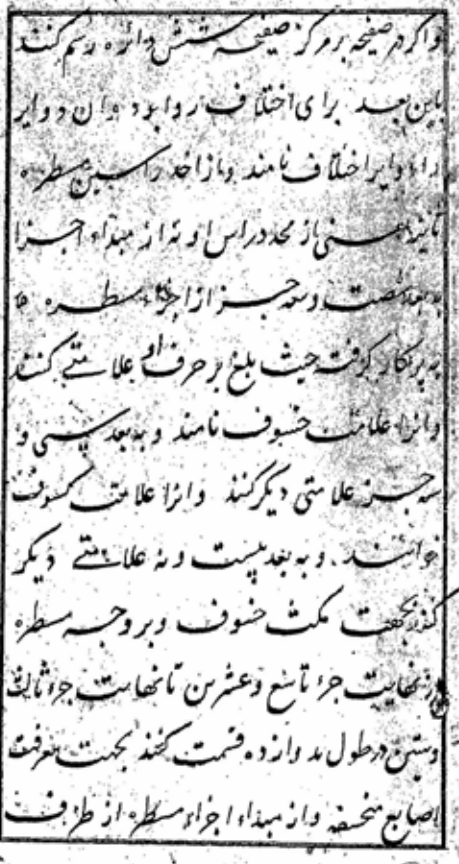
این شکل متعلق است بر نصف قطر

f.12r

first ruler make a (semi)circular depression in it such that the projection on the first, when needed, may enter it, and both rulers become as one ruler. And the numbers of the second ruler are to be written from one side to the other, outward and in opposite directions. And we call the first ruler the alidade, and the second ruler we call the ruler. Then, on one of the two sides of the alidade make six marks for the six planets other than the sun, and we call these the difference marks, and the distances of the marks from the center of the plate for each planet are detailed herewith:

Distances of the difference marks in 60ths of half the plate diameter:					
☾	♄	♃	♂	♀	☿
5;17	5;38	10;38	30;32	42;25	18;13

This distance is to the amount of half the epicycle diameter of these planets.



f.12v

And if on the plate, around the plate center, six circles are drawn with these distances (as radii) for difference(s), it is permissible, and those circles are called difference circles. And from one of the two heads of the second ruler, i.e., from beyond its head, not from the beginning of divisions, to a distance of sixty-three divisions of the divisions of the ruler, laid off with a compass, wherever it reaches, on its edge make a mark, and that is called the lunar eclipse mark. And at a distance of thirty-three divisions make another mark, and it is called the solar eclipse mark. And at a distance of twenty-nine make another mark for the duration (or first totality) of the lunar eclipse, and on the face of the ruler, between the limits of the twenty-ninth division to the sixty-third division, divide in length (the segment thus determined) in twelve (parts) for the determination of lunar eclipse digits. And from the beginning of the divisions of the ruler, on the other side,

دیگر تا نهایت جزء ثالث و ثلثین بدو ازده
قسمت کنند بجهت معرفت اصابع تنکسفه و
اعداد اصابع در بهره و این جهت مرکز بنویسند
و سطر تین را بسلسله دقیقه مرتبط کرده باشند
که طول سلسله قریب باشد بطول نصف قطر
و نصب لبنتین در عضاده در اخذ ارتفاع بکارایند
و اگر بجای سطر ریسمانی باریک استعمال
کنند مطلوب حاصل کرده باشد و بعضی
در رسم جداول اوساط و غیرها بر ظهر صفحه
جدولی رسم کنند که در عرض بیازده قسمت
کرده باشد یکی از جهت صف اعداد و پنج بجهت
اوساط بزبن و علویه و پنج باقی بجهت اوج
شمس و خاصه قمر و وسط جوزهر و

f.13r

to the end of the thirty-third division divide (it) into twelve parts for the knowledge of the solar eclipse digits, and write the numbers of the digits of both, (beginning) from the center. And let the two rulers be joined with a thin chain such that the length of the chain be nearly the length of half the diameter. And the placing of two sights on the alidade may be used for taking the altitude. And if, in place of the ruler, a thin string is used, the desired result may be attained. CHAPTER FIVE On Drawing the Tables of Mean (Motions), etc. On the back of the plate draw a table whose width is divided into eleven parts, one for the column of numbers, and five for the mean (motions) of the sun and moon and superior planets, and the remaining five for the solar apogee, and the lunar anomaly, and the mean of the lunar nodes, and

و خاصهٔ مرکبهٔ سطلین و در طول به پنجاه و هشت
قسم منقسم سه بجهت مطور القاب و دو برای
حرکت اوساط در ده سال متوالی از سنین ناقصه
یزدجردی و نوزده بحسب سنین عشرات و مآت
و الف و سیزده برای شهور اثنی عشر
و خمسه و دوازده برای حاد و عشرات الیاء و یکی برای
ساعت و جدولی دیگر رسم کنند در عرض منقسم
بچهار قسم یک صف بجهت اسماء کواکب
خمسه متحیره و یکی برای مبداء حدود مقامات
رجعت و دیگری منتهای حدود مقامات
رجعت و یکی دیگر برای تفاضل منهما
و ما این جدول را در باب نهم در معرفت
رجعت و استقامت اورديم و اگر این

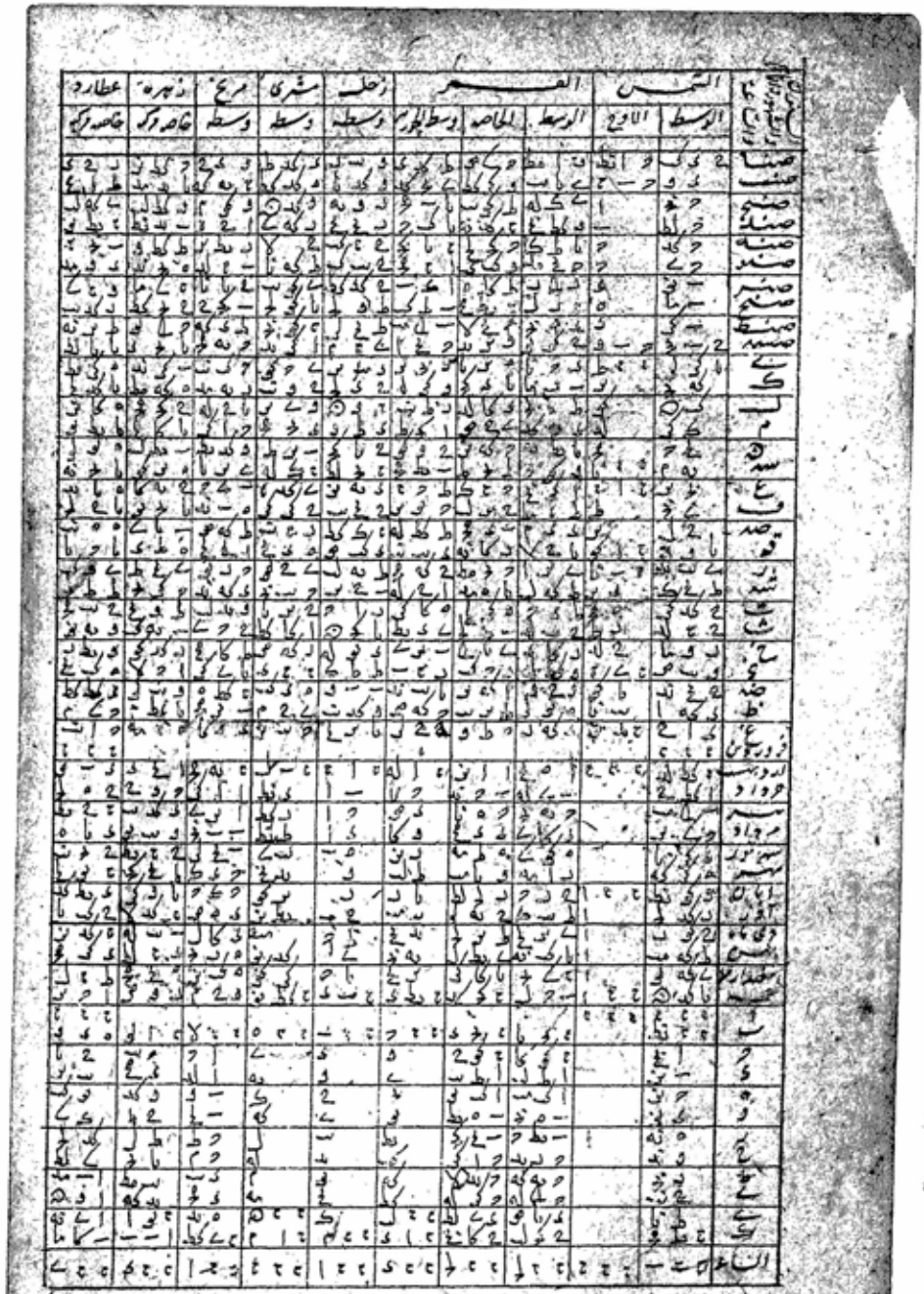
f.13v

the compound anomaly of the two inferior planets; and in length (it) is divided into fifty-eight parts: three for the column headings, and ten for the motion of the mean (planets) in ten successive years of incomplete Yazdigerd years, and nineteen on account of the tens and hundreds and a thousand, and thirteen for the twelve months and five (intercalary days), and twelve for single (days) and tens of days, and one for the hour. And lay out another table (to be) divided into four parts, one row for the names of the five planets, and one for the beginning of the limits of the retrograde stations, and another (for the) ends of the boundaries of the retrograde stations, and one more for the differences between the two of them. And we have brought forth this table in the ninth chapter, on the determination of retrogradations and stations, and if this

جدول راه کم میکنند و بر اجزاء حجر برای
مبادی حد و استقامت و نهایات ان علامات
نهند بهتر و یک جدول دیگر وضع کنند
بجهت اختلاف ساعات و این اجتماع
حقیقی و مرئی بود و اختلاف منظر اثر
در عرض عند الاجتماع المرئی بعرض
وسط اقالیم و ما این جدولها را از زیج خاقانی
عمل کرده نوشتیم و اگر این جدولها را در آله
وضع نکنند و عند الاحتیاج این رساله
بردارند جایز است و جدول اختلاف ساعات
چون در معرفت کسوف بکار می آید در باب
معرفت کسوف نوشتیم و صورت جدول
اوساط اینست و الله اعلم

f.14r

table is not drawn, and on the divisions of the ring marks are put for
the beginning of the boundaries of the stations and their ends,
(it will be) better. And place another table
for the difference of the hours, and that will be between the
true and apparent conjunction, and the lunar parallax
in latitude according to the apparent conjunction at the
mean latitude(s) of the climates. And we, having made (?) these tables in the Khāqānī Zīj,
wrote them (from it). But if these tables are not
put on the instrument, and, when necessary, are taken from this treatise,
it is permissible. And the table of difference of the hours,
since it was being used in the science of solar eclipses, we wrote it in the chapter (on)
the science of solar eclipses. And the picture of the table of
mean motions is this, but God knows better.



TRANSLATION

f.14v (A Table of Mean Positions and Mean Motions)

Years, months, days, & hours	☉ Mean	☉ Apogee	☾ Mean	☾ Anomaly	☾ ☊	♄ Mean	♃ Mean	♂ Mean	♀ Compound Anomaly	☿ Compound Anomaly
851	8° 4;22″	3° 1;59′	6° 1;49′	3° 28;46′	9° 24;4°	6° 12;7′	4° 24;9′	6° 4;8′	3° 29;57′	7° 8;4°
852	4;7	3 2;0	10 11;12	6 27;29	10 13;24	6 24;[2]1	5 24;29	0 15;25	11 14;44	9 1;48
853	3;53	1	2 20;35	9 26;12	11 2;43	7 6;3[5]	6 24;50	6 26;42	6 29;32	10 25;32
854	3;39	2	6 29;58	0 24;55	11 22;3	7 18;4[8]	7 25;10	1 8;0	2 14;[1]9	0 19;16
855	3;24	3	11 9;20	3 23;38	0 11;23	8 [1;2]	8 [25;3]1	7 19;17	9 29;6	2 13;0
856	3;10	3	3 18;44	6 22;21	1 0;43	8 [3;16]	9 25;51	2 0;34	5 13;54	4 6;44
857	2;56	4	7 28;7	9 21;5	1 20;2	8 [25];19	10 26;12	8 11;51	0 28;41	6 0;28
858	2;41	5	0 7;30	0 19;49	2 9;22	9 [7;4]3	11 26;33	2 23;8	8 13;29	7 24;12
859	2;27	6	4 16;53	3 18;31	2 28;42	9 [9;5]7	0 26;53	9 4;25	3 28;16	9 17;55
860	8 2;13	3 2;0	8 26;16	6 17;14	3 18;1	10 [2;10	1 27;14	3 15;43	11 13;4	11 11;39
10	11 27;31	0 0;9	7 3;51	5 17;11	[6] 13;17	4 2;17	10 3;26	3 22;52	2 27;54	5 27;19
20	25;13	17	2 7;[2]	11 4;23	0 26;35	8 4;33	8 6;52	7 15;44	5 25;49	11 24;38
30	22;50	26	9 11;33	4 21;34	7 9;52	[0] 6;50	6 10;17	11 8;35	8 23;43	5 21;27
40	20;26	34	4 15;24	10 8;46	1 23;9	4 9;6	4 13;43	3 1;27	11 21;38	11 19;16
50	18;3	43	11 19;15	3 25;57	8 6;26	8 11;23	2 17;9	6 24;19	2 19;32	5 16;56
60	15;40	0 0;51	6 23;4	9 13;9	2 19;43	0 13;39	0 20;35	10 17;11	5 17;26	11 13;55
70	13;17	0 1;0	1 26;58	3 0;20	9 3;0	4 15;56	10 24;0	2 10;3	8 15;21	5 11;14
80	10;53	9	9 0;49	8 17;32	3 16;17	8 18;12	8 27;26	[6] 2;54	11 13;16	11 8;33
90	8;30	17	4 4;40	2 4;43	9 29;35	0 20;29	7 0;52	9 25;46	2 11;10	5 5;52
100	11 6;6	0 1;26	11 8;31	7 21;55	4 12;52	4 22;46	5 4;18	1 18;38	5 9;4	11 3;11
200	10 12;13	0 2;51	10 17;1	3 13;49	8 25;43	9 15;32	10 8;36	3 7;16	10 18;9	10 6;22
300	9 18;20	4;17	9 25;32	11 5;44	1 8;35	2 8;17	3 12;53	4 25;54	3 27;13	9 9;33
400	8 24;27	5;43	9 4;3	6 27;36	5 21;27	7 1;3	8 17;11	6 14;32	9 6;18	8 12;[4]4
500	8 0;34	7;9	8 12;34	2 19;33	10 4;19	11 23;50	1 21;29	8 3;10	2 15;22	7 15;56
600	7 6;41	8;34	7 21;5	10 11;26	2 17;10	4 16;35	6 25;47	9 21;48	7 24;27	6 19;7
700	6 12;47	0 10;0	6 29;35	6 3;22	7 0;2	9 9;20	0 0;4	11 10;27	1 3;31	5 22;18
800	[5] 18;54	11;26	6 8;6	1 25;17	11 12;54	2 2;6	5 4;22	0 29;5	6 12;36	4 [5]5;29
900	4 25;1	12;51	5 16;37	9 17;12	3 25;46	6 24;52	10 8;40	2 17;43	11 21;40	3 28;40
1000	4 1;8	0 14;17	4 25;7	5 9;6	8 8;37	11 17;38	3 12;57	4 6;21	5 0;45	3 1;52
Farvardīn	0 0;0									0 0;0
Ordībehešt	0 29;34	0 0;0	1 5;18	1 1;57	0 1;35	0 1;0	0 2;30	0 15;43	1 18;4	4 2;46
Khordād	1 29;8		2 10;35	2 3;54	3;11	2;1	4;59	1 1;27	3 6;8	8 5;33
Tīr	2 28;42		3 15;53	3 5;51	4;46	3;1	7;29	1 17;10	4 24;12	0 8;39
Mordād	3 28;17		4 21;10	4 4;48	6;21	4;1	9;59	2 2;53	6 12;16	4 11;5
Shahrīwer	4 27;51		5 26;[27]	5 9;45	7;57	5;2	12;28	2 18;37	8 0;19	8 13;32
Mehr	5 27;25		7 1;45	6 11;42	9;32	6;2	14;58	3 4;20	9 18;23	0 16;38
Ābān	6 26;59	0 0;1	8 7;3	7 13;38	11;30	7;2	17;27	3 20;3	11 6;27	4 19;24
Ādhar	7 26;33	1	9 12;20	8 15;36	12;42	8;2	19;57	4 5;47	0 24;31	8 22;11
Dey Māh	8 26;7	1	10 17;36	9 17;33	14;18	9;3	22;26	4 21;30	2 12;35	0 24;57
Bahmen	9 25;42	1	11 22;55	10 19;30	15;53	10;3	24;56	5 7;13	4 0;39	4 27;43
Isfandārmadh	10 25;16	1	0 28;13	11 21;27	17;18	11;3	27;26	5 22;57	5 18;43	9 0;30
Five days	11 24;50	0 0;1	2 3;30	0 23;24	0 19;4	0 12;4	0 29;56	5 8;40	7 6;47	1 3;16
1	0;0	0 0;0								0 0;0
2	0 0;59		13;11	0 13;4	0 0;3	0 0;2	0 0;5	0 0;31	0 1;36	0 4;6
3	0 1;58		0 26;21	0 26;8	6	4	10	1;3	3;12	8;11
4	0 2;57		1 9;32	1 9;12	10	6	15	1;34	4;48	12;17
5	0 3;57		1 22;42	1 22;16	13	8	20	2;6	6;24	16;22
6	0 4;56		2 5;53	2 5;19	16	10	25	2;38	8;1	20;28
7	0 5;55		2 19;3	2 18;23	19	12	30	3;9	9;37	24;33
8	0 6;54		3 2;14	3 1;27	22	14	35	3;40	11;13	28;39
9	0 7;53		3 15;25	3 14;31	25	16	40	4;12	12;49	1 2;44
10	0 8;52		3 28;35	3 27;35	29	18	45	4;43	14;25	1 6;50
10	0 9;51		4 11;46	4 10;39	0 0;32	20	0 0;50	5;14	0 16;1	1 10;55
20	0 19;45		8 23;32	8 21;58	0 1;4	0 0;40	0 1;40	0 10;29	1 2;2	2 21;51
The hour	0 0;2	0 0;0	0 0;33	0 0;33	0 0;[0]	0 0;[0]	0 0;0	0 0;1	0 0;4	0 0;10

خاتمه در صفت لوح اتصاف الاسطرلاب و
صورت آن بگیرند لوحی از چوب صلب یا
یا صفر که طولش یک ذراع و عرضش بیست
ملتوان ذراع و بر راست وسط لوح بقدر ما ممکن
کنند و بر دو جانب لوح مثلثی قایم الزاویه رسم
کنند چنانکه هر یک از دو ضلع زاویه قایمه
مثلث بوتری دو ضلع لوح بود کل نظیر بعد
ضلع پست ضلع اطول زاویه قایمه را که قاعده
مثلث است بیست و چهار قسم کنند بر ابر
ساعات و هر قسمی را شصت ماهان قدر
که توانند بجانب صفر کبر لوح قسمت کنند
و ضلع بلند را قسمت کرده هر قسمی را بشصت
یا آنچه توانند قسمت کنند و اگر اقسام بیرات

TRANSLATION

f.15r

CONCLUSION, On the Construction of the Plate of Conjunctions. And (so) get its picture; a plate of hard wood, or yellow copper, or brass, that its length be one cubit and its width more than two thirds of a cubit. And, to the extent possible, attempt to make the surface of the plate flat. And on the face of the plate draw a right triangle such that each one of the two legs of the triangle be parallel to two sides of the plate respectively, at a proper distance. Then the longer leg of the triangle, which is the base of the triangle, divide into twenty-four parts for the hours, and each part is divided into sixty (parts) or to the amount which it can be (divided), taking into account the smallness or largeness of the plate. And the *shorter* leg, having been divided into sixteen parts, each part is divided into sixty, or to the extent to which it can be divided. And if the divisions of the (planetary) paths

The method of verifying the right angle is this, with a compass from one of the two sides of the right angle lay off three equal parts, and from the other side, with just the same opening of the compass, lay off four parts, and from the end of the third part to the end of the fourth part join a line so that it is the chord of the angle. If that line, with the same opening of the compass is five parts (the angle is verified), and otherwise it is not a right (angle). Its proof is that the squares on the two sides of the triangle are equal to the square of the chord of the right angle of that triangle, as is explained in (the Theorem of) the Bride (the Pythagorean Theorem).

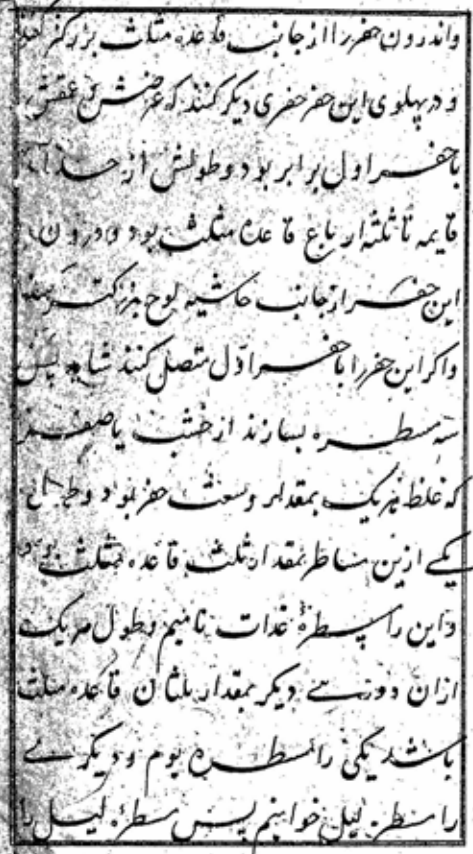
I.e., in the width of the plate have that amount outside the triangle that the number(s) of the parts of travel (can) be written, and in the length have that amount that the rulers and numbers of hours may find place.

*Marginal correction.

باقسام ساعات برابر باشد هر یک
وحودی بهتر بود استخراج لاوجو باپس
از هر قسمی از اقسام صلیبی خطوط اخراج کنند
موازی ضلع آخر تا متصل شود بخطی
که از اقسام آن ضلع دیگر موازی این ضلع
اخراج کرده باشد و تمیز میان خطوط خارجه
از مقاسم ساعات و اجزاء سرات و خطوط
خارجه از دقایق هر یک بالوان مختلفه
کنند و همچنین حصات اجزاء دقایق را
نیز بلونی مغایر آنها تمییز کنند که عند العمل
آسان کرده و در خارج ضلع اطول مثلث
حفری کنند سر تا بسر لوح که عرضش بقدر
اصبعی باشد و عمقش بمقداری صالح بود و

f.15v

be equal to the divisions of the hours, in largeness
and smallness (of divisions), it will be better; adornment is unnecessary. Then,
from each of the divisions of the two sides, extend lines
parallel to the other side until they join the line(s)
which have been extended from the divisions of that other side parallel to this (first) side.
And (for) distinguishing between the lines extended
from the divisions of hours and parts of (planetary) paths, and the lines
extending from the minutes, make each in different colors.
And in the same fashion, the fifths of parts and minutes
also distinguish them by a color differing from those (others), that operation (with it)
be facilitated. And outside the longer side of the triangle
make a trough from one side of the plate to the other, that its width be to the amount of a
digit and its depth be to a proper amount, and



f.16r

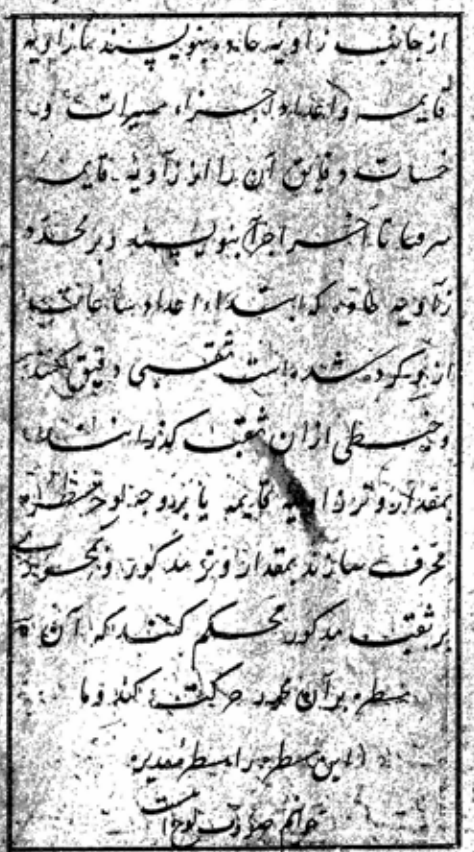
make the inside of the trough bigger than (it is) beside the base of the triangle.
And beside that trough, make another trough such that its width and depth
be equal to (those of) the first trough, and its length
(as compared to) the limit of that right *angle* of the triangle should be three fourths the base of the triangle, and the inside
of this trough is made bigger than the (width) alongside the margin of the plate.
And if this trough is joined with the first trough, perhaps (it will be satisfactory). Then
make three rulers, of wood or brass,
that the thickness of each one to be the amount of the width of the trough, and the length of
one of these rulers be to the amount of a third of the base of the triangle.
And we call this the next-day ruler, and let the length of each
of these other two be to the amount of two thirds of the base of the triangle.
One of these we call the day ruler, and the other
the night ruler. Then put the night ruler

*Marginal addition.

در حفر مالی و سطر ه غدات و یوم
را در حفر اول کنند بوضعی که سطر ه
غدات از جانب زاویه فایده بود و این
سه سطر در این حفرها حرکت کنند از وجه
لوح مرتفع نگردند و سطوح ظاهر سه سطر
با سطح لوح بمثابه یک سطح مستوی گردد
و دو سطر را باجرای قاعده مثلث بر
دو قابض قسمت کنند پس سطر ه غدات بهشت
نهم و آن دو دیگر شانزده قسمی بود
و ارقام اعداد بر سطوح از جانب زاویه
حاده بسوی زاویه قائمه تویسند
و در مابین حفر و محیط لوح اعداد ساعات
و خسات دقایق قاعده مثلث را

f.16v

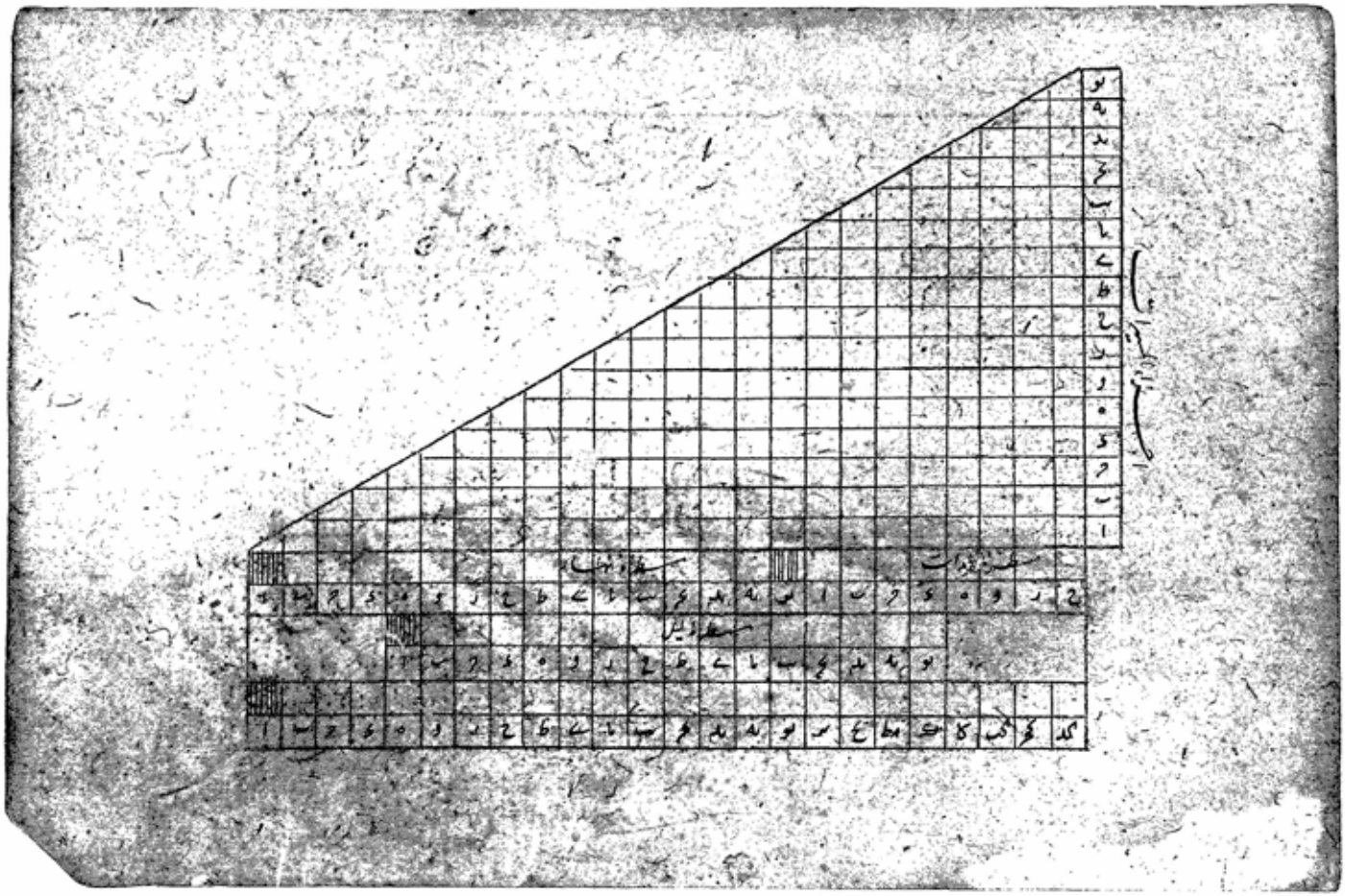
in the second trough, and the next-day and day rulers
put in the first trough in such fashion that the
next-day ruler be from the side of the right angle. And these
rulers must be able to move (i.e. slide) in these troughs, but should not be (capable of being) raised above the surface of the
plate, but the visible surfaces of the rulers
should be like a single plane surface.
And divide the face(s) of the ruler(s) into the divisions of the base of the triangle and
its minutes. So the next-day ruler (will be divided) into eight
parts, and those other two (into) sixteen, and
write the signs of the numbers of each ruler from the side of the
acute angle along the base of the triangle.
And between the trough and the perimeter of the plate, write the numbers of the hours
and of the fives of minutes of the base of the triangle



از جانب زاویه عاده بنویسند مانند زاویه
قایمه و اعداد اجزاء میرات و
خسارات و فایض آن را از زاویه قایمه
سر وقا تا آخر اجرا بنویسند و بر محدد
زاویه طاقه که بست از اعداد ساعات
از یکدیگر دیشد باشد تقسیمی دقیق کنند
و خطی از آن شعب کذرانند
بمقدار دو قرد زاویه قایمه یا بردو جانب خط مستقیم
محرف سازند بمقدار او نزدیک مذکور و بخطوط
بر شعب مذکور محکم کنند که آن
خطوط بر آن محدد حرکت کنند و ما
این خط را خط میدیر
خوانیم صورت لوح است

f.17r

from the side of the acute angle to the right angle. And write the numbers of the divisions of (planetary) paths, and the fives of their minutes from the right angle from beginning to end. And at the extremity of the acute angle, which is from (where) the beginning of the numbers of the hours, has been made, drill a minute hole. And make a thread to pass through that hole, (in length) to the amount of the hypotenuse of the right triangle. Or, on the face of the plate emplace an edged ruler, to the amount of the (above-) mentioned hypotenuse, and fasten it to an axis on the (above-)mentioned drill(-hole) such that that ruler move about that axis. And we call this ruler the turning ruler. And the picture of the plate is this:



f.17v

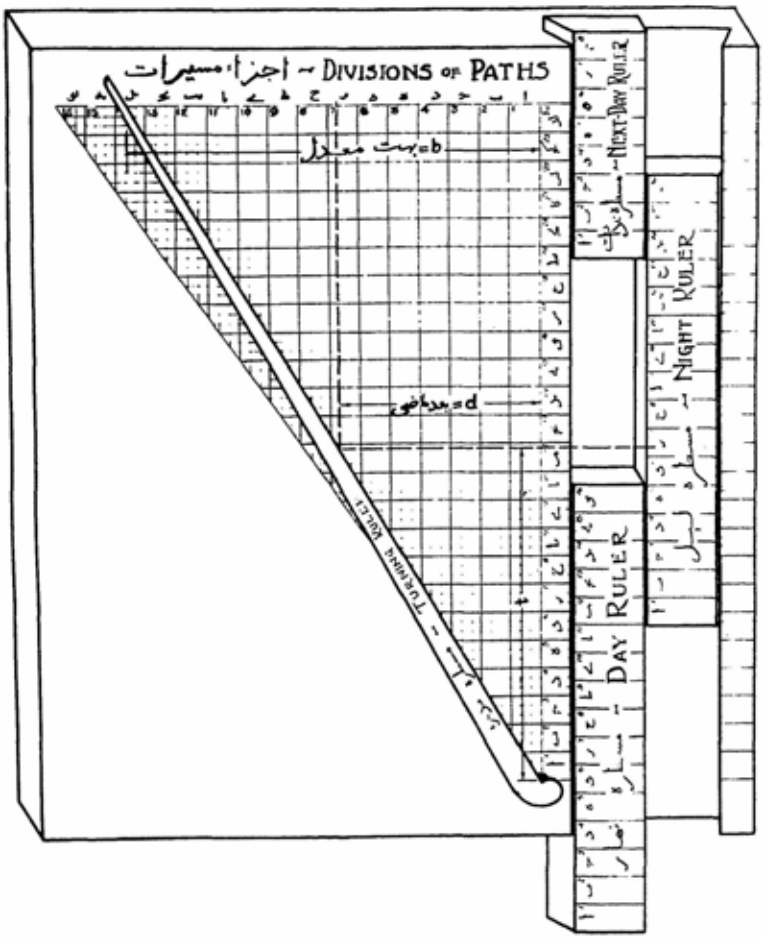


Figure 2. The Plate of Conjunctions

مقاله دوم در عمل بطلمیوس

مشتمل بر باب و خاتمه باب اول

در ترتیب اوج شمس اگر بر صفحه کرده بود

بر محاذی اوج شمس نقطه از اجزاء جنبر

یعنی بر موضع شمس که از جداول گرفته اند نهند

و سیار را در احد اثقاب فرو برند و چون زمانی

بگذرد که اوج از موضع خود حرکت کرده باشد

سیار را برآورده باز همان حرکت کنند

باب دوم در استخراج اوساط کواکب

وقتی که خواهند اوساط کواکب در آن وقت

استخراج کنند ان وقت را از تاریخ فارسی

معلوم کنند اگر در سالهای ناقصه مبسوطه باشد

بعینه آنچه در ازای آن سال نوشته شده برای هر کوکب

f.18r

1 THE SECOND TREATISE:
On the Operation of the Plate of Heavens,
2 Containing Fifteen Chapters and a Conclusion.
CHAPTER ONE:
3 On the Arrangement of the Instrument. The apogee of the sun, which had been put on the plate,
4 put opposite the apogee of the sun on the divisions of the ring,
5 i.e. put it on the position of the sun which has been obtained from the table.
6 And stick the peg into one of the holes, and when a time
7 passes, so that the apogee should have moved from its position,
8 the peg having been withdrawn, give (the plate) that motion,
9 CHAPTER TWO: On the Extraction of the Mean (Positions).
10 Whenever it is desired to extract the mean (positions) of the planets at that (i.e. at such and such a) time,
11 determine that time from the Persian calendar.
12 If it is in the explicit years (of the table),
13 take exactly what is written opposite that year for each planet.

We have set the beginning of the mean motions at the incomplete (or explicit Yazdigerd) year 851, which is the first of the year of the enthronement of His Majesty the sovereign upon the seat of rulership, may God strengthen the throne of the kingdom by its enduring until the Day of Judgment, and increasing the marks of his justice and presence for all time! Amen, O Lord of the Worlds!

f.18v

But if it be before or after, take opposite a year (such) that between it *and the desired year* there be to the amount of one of (the tabulated) sum(s) of complete years. And that which is opposite that incomplete year to that which has been taken opposite that complete year, add if the desired year is after the incomplete years (tabulated). Otherwise that which was obtained opposite the complete years, and that which was obtained opposite the incomplete years, subtract (them). Then, that which is found opposite the desired month and day from the table of months and days, add to the result (obtained for) the first of the year so that the mean (position) in the desired year and month and day result, for noon at the longitude of Constantinople, may she preserve in safekeeping her lord from calamities and affliction, and that is 60;0°. And if (it) is desired at a longitude other than 60°, (the amount) between the two longitudes having been made (into) hours and minutes, whatever it become, the result of

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* Marginal addition.

حرکت یکساعته را که در جدول نهاده ایم در آن
ساعت و دقایق ضرب کنند و حاصل ضرب را
بر حاصل وسط در نصف النهار سه زیاده کنند
اگر طول مطلوب کمتر باشد و نقصان کنند اگر بیشتر
بود و همچنین اگر در وقتی غیر نصف النهار
خواهند ساعات و دقایق گذشته از نصف
النهار یا باقی تا نصف النهار را در حرکت وسط
در یکساعت ضرب کنند و حاصل را بر حاصل
مذکور افزایند اگر بعد از نصف النهار بود و نقصان
کنند اگر قبل از نصف النهار باشد حاصل وسط
مطلوب بود در وقت مطلوب و وسط
شمس بعینه وسط سفلیین است و این مساوی
وسط و خاصه هر یک از علویه است یعنی

f.19r

the motion (for) one hour, which we have put in the table, multiply by that (number of)
hours and minutes, and increase by the product
the result of the mean (motion) for the noon of (longitude) 60
if the desired longitude is less, and decrease (it) if (it) is more.
And in the same manner, if (it) is wanted at a time other than noon,
the hours and minutes passed beyond noon,
or those remaining until noon, multiply by the motion of the mean
in one hour, and the result is added to the
(above-)mentioned result if (it) is after noon, and decrease (it)
if it is before noon, the result will be the
desired mean at the desired time. And the mean of the
sun is exactly the mean of the two inferior (planets), and this is equal to
the mean anomaly of each of the superior (planets), i.e., it is

مساوی خاصه مرکبه ایشانست وچون وسط آفتاب
بسینه وسط سفلیین است وسط ایشانرا در جدول
نهادیم وبجاوی وسط خاصه مرکبه ایشانرا نوشتیم

ایم باب سیوم در معرفت تقویم

آفتاب وتعدیل او وبعدش از مرکز عالم رمثل
وسط از اجزاء دجره علامتی نهند ومانرا علامت
وسط خوانیم پس حرف مسطره را بر علامت
وسط ومرکز مستعار بگذرانند چنانکه مبدا اقسام
مسطره بر علامت وسط افتد وحرف عضاد را
بموازی مسطره گردانند یعنی دو قوس که واقع
بود میان المسرطتین از دو طرف متساوی کرده
بر موضع عضاد از اجزاء دجره بقرب علامت
وسط از اجزاء دجره علامتی نهند وانرا موضع مقوم

f.19v

equal to their compound anomaly. And since the sun's mean
is exactly the mean of the inferior (planets), we have not put their mean (longitudes) in the table,
and in place of it we have written their compound anomalies.
CHAPTER THREE. On the Determination of the True Longitude of the
Sun, and Its Equation, and Its Distance from the Center of the Universe. According to (the sun's)
mean (longitude) put a mark on the divisions of the ring, and this we call the mark of the
mean. Then, make the edge of the ruler to pass alongside the mark of the
mean and the fictitious center so that the beginning of divisions of the
ruler fall on the mark of the mean. And (then) the edge of the alidade
is made parallel to the ruler, i.e., the two arcs which are formed
between the two rulers having been made equal.
And at the position of the alidade at the divisions of the ring, near the mark of the
mean at the divisions of the ring, a mark is placed and that we call the true position.

This is in order that the distance of the sun from the center of the universe can be ascertained.

خوانیم از اوّل حمل تا موضع مقوم تقویم نبود و ما بین علامت
وسط و موضع مقوم تعدیل باشد و میان مرکز مستعار
و علامت وسط از اجزاء مسطره بعد آفتاب
از مرکز عالم باجزایی که نصف قطر خارج مرکز
باشد والله اعلم باب چهارم
در معرفت تقویم ماه چون وسط شمس را از وسط قمر
نقصان کنند بعد ماند و چون تضعیف بعد کنند مرکز
قمر معلوم گردد و ازا بعد مضاعف خوانند پس مرئی
عضاده را بر مثل مرکز قمر از اجزاء مجره بخانه
بر موضع تقاطع حرف عضاده بمنطقه قمر علامت
کنند این علامت مرکز قمر بود پس حرف مسطره
را بر نقطه محاذاة و علامات مرکز نهند
و عضاده را موازی مسطره کنند و بر موقع مرئی عضاده

f.20r

From the first (point) of Aries to the true position will be the true longitude, and (the angular distance) between the mark of the mean and the true position is the equation. And between the ficticious center and the mark of the mean, (measured) in divisions of the ruler is the sun's distance from the center of the universe, in such divisions that half the external (i.e., deferent) diameter is sixty, but God knows better. CHAPTER FOUR On the Determination of the True Longitude of the Moon. If the sun's mean is taken from the moon's mean a distance remains (the mean elongation), and if the distance is doubled the center (of the deferent) of the moon becomes known, and this is called the doubled distance (the double elongation). Then having put the pointer according to the moon's center on the divisions of the ring, at the place of intersection of the edge of the alidade with the moon's deferent, make a mark. Let this be the mark of the moon's (epicycle) center. Then put the edge of the ruler along the opposite point and the mark of the center, and make the alidade parallel the ruler, and at the place where the pointer of the alidade has fallen

از اجزاء جزء علامتی کنند و این مبدا حرکت
خاصه بود و بعد از ان عضاده را بر خلاف توالی
حرکت دهند بمقدار حرکت خاصه قمر یعنی خاصه
مرجعه بروج و درجه و دقیقه که بود از ان مبدا و آن قدر
بروج و درجه بشمارند هر جا که منتهی شود بورقم
اختلاف علامتی کنند و لابحال بر خلاف جهت
منتهی حرکت خاصه بود و این علامت اختلاف
قمر بود و بعد ذلک حرف سطره را بر هر دو علامت
مرکز اختلاف نهند و عضاده را موازی گردانند
بر موقع هری عضاده بقرب علامت مرکز علامتی
کوفند بر اجزاء جزء و این را موضع معدوم نامیم پس
فضل وسط قمر را بر هر مرکز بر مابین اول حمل و موضع
معدوم زیاده کردانند تقویم حاصل شود و الله اعلم

f.20v

make a mark at the divisions of the ring. And this is the beginning of the motion of the anomaly. And after this move the alidade contrary to the succession (of the zodiacal signs) to the amount of the lunar anomaly, i.e., the anomaly however many signs and degrees and minutes it be, from this beginning, count off that many signs and degrees. Wherever it ends, at the mark of the difference, make a mark. And make sure it is opposite the direction of the end of the anomalistic motion, and let this be the difference mark of the moon. And after this, the edge of the ruler is placed along both the mark of the center and the difference, and make the alidade parallel (to the ruler). At the position of the pointer of the alidade near the mark of the center make a mark on the divisions of the ring, and this we call the true position. Then the excess of the moon's mean over the center between the first (point) of Aries and the true position, let (it) be increased. The true longitude results, but God knows better.

باب پنجم در معرفت تقویم خمسه متحیره

مری عضاده بر مثل وسط نهند از اجزای مجموعه
و طرف مسطره را بر نقطه معدل للسیر گذرانیده
بموازی عضاده سازند پس بر موضع تقاطع طرف
مسطره با منطقه بر کوکب علامت کنند آن علامت
مرکز آن کوکب بود پس مری را با عضاده بر
بر وسط شمس نهند بجهت هر یک از کواکب
علویه سوی بر رقم اختلاف هر یک علامتی
کنند بر صفحه این علامت اختلاف طولی
آن کوکب بود و لا محاله علامات اختلاف
علویه بر خطی وسمی باشد که واصل بود میان
مرکز و نظر وسط آفتاب و بجهت سفلیین مری
عضاده را بر خاصه مرکبه هر یک نهند و بر رقم

f.21r

CHAPTER FIVE. On the Determination of the True Longitude of the Five Planets.

Place the pointer of the alidade according to the mean (longitude) on the divisions of the ring,

and, having made the edge of the ruler to pass alongside the equant,

make it parallel the alidade. Then, at the place of intersection of the edge of the

ruler with the deferent of each planet make a mark. This shall be the

center mark of that planet. Then put the pointer of the head of the alidade

at the mean of the sun in the case of each one of the

superior planets, and at the difference mark of each make a mark

on the plate. This will be the longitudinal difference mark of

that planet, and make sure that the difference mark (in the case) of a

superior (planet) is always along a line which joins

the center to the opposite (i.e., the supplement) of the sun's mean (longitude). But as for the inferior (planets), the pointer of the

alidade is put along the compound anomaly of each one, and at the mark of the

اختلاف بر صفحه علامتی نهند و این علامت
اختلاف ان کوکب بود و این نیز در جهت
نقطه خاصه مرکبه واقع خواهد شد و بعد ذلک حرف
سطرنج را بر علامت مرکز و اختلاف هر کوکب
کذرانند بر موقع مری عضاده که بقرب علامت
مرکز بود علامتی نهند و آن موضع بمقوم و از اول
حمل تا موضع بمقوم تقویم آن کوکب بود والله اعلم

کذرانند و حرف عضاده را مواری آکرداند

باب ششم در معرفت تعدیلات و معرفت
مرکز و خاصه معدله اگرچه در استخراج تقویم باین
ابواب احتیاج باین تعدیلات ندارد لکن چون خواهیم
کرد بدانیم بر مثل وسطی نهر کوکبی بر حجره علامتی کنند
و حرف عضاده را بر علامت مرکز کذرانده بر
مری عضاده بقرب وسط علامتی دیکر کنند بر حجره

f.21v

difference put a mark on the plate. And this is the mark of the
difference of that planet, and this also will befall in a direction
opposite to that of the compound anomaly. And after that, make the edge of the
ruler pass along the mark(s) of the center and the difference of each planet,
and put the edge of the alidade parallel to the ruler. At the position of the pointer of the alidade which is near the mark of the
center a mark is put, and that will be the true position, and from the first (point) of
Aries to the true position will be the true longitude of that planet,
but God knows better. CHAPTER SIX. On the Determination of Equations, and the Determination
of the Center, and the Adjusted Anomaly. Although for the extraction of the true longitudes with this
instrument we do not need these equations, nevertheless if we want
to know (them), according to the mean of each planet, a mark is put on the ring,
and, having put the edge of the alidade alongside the mark of the (epicycle) center, at the
pointer of the alidade near the mean (i.e., on the same side as the mean), make another mark. On the ring,

*Marginal correction.

مابین العلامتین از اجزاء تعدیل شمس و تعدیل
اول متحیز بود و تعدیل اول قسم بقدر مابین
علامت ثانی و علامت مبدا حرکت خاصه بود
و تعدیل ثانی متحیزه بقدر مابین علامت ثانی و موضع
مقوم ان کوکب بود و موضع علامت ثانیه وسط
معدل شمس و هر یک از کواکب متحیز باشد
و چون اوج هر کوکب را از وسط معدل او نقصان
کنند مرکز معدل آن کوکب باقی ماند و چون وسط
معدل هر یک از علویه را از وسط شمس و وسط
معدل هر یک از سفلیه را از خاصیه مرکبه او نقصان
کنند خاصه معدل آن کوکب باقی ماند والله اعلم

باب هفتم در معرفت عروض کواکب

ستة بجهت قمر وسط چون خواهد را بر تقویم قمر افزاید

f.22r

between the two marks (measured) in the divisions of the ring will be the equation of the sun and the
first equation of the planet(s). And the first equation of the moon will be to the amount between
the second mark and the mark of the beginning of the anomalistic motion.
And the second equation of a planet will be to the amount between the second mark and the
true position of that planet. And the place of the second mark, let it be the
adjusted mean of the sun and any one of the planets.
And if the (longitude of the) apogee of each planet be subtracted from its adjusted mean,
the adjusted center of that planet will remain. And if the
adjusted mean of each of the superior (planets) be subtracted from the sun's mean, and the
adjusted mean of each of the inferior (planets) from its compound anomaly,
the adjusted anomaly of that planet remains, but God knows better.
CHAPTER SEVEN. On the Determination of the Latitudes of the Six Planets.
In the case of the moon, increase the mean of the nodes by the true longitude of the moon;

حصه عرض حاصل شود پس مری عضاده را بر مثل
حصه عرض یا بر نظیر حصه عرض نهند از اجزاء حجره
و نظر کنند که نقطهٔ تقاطع حرف عضاده با دایرهٔ
عرض قمر بر چند خط واقع شده است از خطوط عرض
آن قدر عرض بود بس حصهٔ عرض اگر کمتر از شش
درج باشد عرض او شمالی بود و اگر بیشتر از شش
درج باشد عرض او جنوبی بود و بجهت عرض علویه
و عرض ثانی سفلیین بر مرکز معدل هریک از کواکب
علویه ماسین اوج و راس آن کواکب را بیفزایند

ماسین اوج کوکب علوی و راس آن		
زحل	مشتری	مریخ
قم ۱	ع ۱	صه ۱

و حاصل را مرکز عرض نامیم و در سفلیین مرکز

f.22v

the argument of the (lunar) latitude will result. Then the pointer of the alidade is put according to the argument of the latitude, or corresponding to the argument of the latitude on the divisions of the ring. And observe that the point of intersection of the edge of the alidade with the circle of latitude of the moon, on how many lines it has fallen, of the latitude lines. That will be the amount of the latitude. Then the argument of the latitude, if it is less than six signs, its latitude will be northern; and if it is more than six signs its latitude will be southern. As for the latitude of the superior (planets) and the second latitude of the inferior planets, increase by the adjusted center of each one of the superior planets between the apogee and the ascending node of that planet,

Between the apogee of a superior (planet) and its ascending node:		
♄	♃	♂
140;0	70;0	95;0

and the result we call the center of latitude. And with the inferior planets, with the

معدل عمل را تمام کنیم پس سری عضاده را بر مثل
خاصه معدله از اجزاء حجر نهند و بر صفحه بر رقم
اختلاف علامتی کنند و این را علامت اول نامیم
پس عضاده را بر قطر استوا قائم کردانند و حرف
سطر را بر علامت اول گذرانند. موازی عضاده
سازند و بر تقاطع حرف سطر و قطر استوا علامت
دیگر کنند و این را علامت ثانیه خوانیم پس حرف عضاده
را بر قطر استوا منطبق کردانند و علامت ثانیه را بر
عضاده. نقل کنند و عضاده را بقدر غایت میل قسطر
مار بدرود و حضیض هر کوکب از چندا اجزاء بگردانند

غایت میل قطر مار مدرج و حصص

f.23r

adjusted center we finish the operation. Then the pointer of the alidade is put according to the adjusted anomaly (measured) in the divisions of the ring. And on the plate by the mark of the difference a mark is made, and this we call the first mark. Then let the alidade be erected perpendicular to the equating diameter, and, the edge of the ruler having been put beside the first mark, make it parallel the alidade. And at the intersection of the edge of the ruler and the equating diameter let another mark be made, and this we call the second mark. Then the edge of the alidade is turned so that it coincides with the equating diameter, and the second mark is transferred to the alidade. And the alidade is turned by the amount of the maximum inclination of the (epicyclic) diameter passing (through) the epicyclic apogee and perigee of each planet, (measured) from the beginning of divisions.

Maximum inclination of the (epicyclic) diameter passing (through) the (epicyclic) apogee and perigee				
♋	♃	♂	♀	☿
4;30°	2;30°	2;15°	(-)2;30°	6;15°

پس بدو صفحه بر موقع علامت ثانیه که بر حرف عضاد
کرده بودند علامتی کنند آنرا علامت ثالثه نامیم پس
همه برای علویه مری عضاده را بداخل غایت میل
مایل از ممتل نهند و حرف مسطره را بر علامت

غایت میل مایل کواکب از ممتل					
زحل	مشتری	مریخ	شمس	زهره	عطارد
۲۴	۱۱	۱۹	۲۰	۲۷	۲۲

ثالثه گذرانده موازی عضاده سازند و بر صفحه از
علامت ثالثه بر موقع حرف مسطره به بعد رقم
اختلاف آن کوکب در جهت مبدا اجزای میاه
خطی کشیده و این خط را خط میل نامیم و بجهت یافتن
حرف مسطره را بر علامت ثالثه نهاده مسطره را
موازی قطر استوا کرده اند و از علامت ثالثه

f.23v

Then, on the plate, at the place of the second mark, which had been made on the edge of the alidade, a mark is made, and that we call the third mark. Then, for the superior (planets), the pointer of the alidade is put according to the maximum inclination of the inclining (deferent plane) from the parecliptic, and, the edge of the ruler having been placed along the

Maximum inclination of the inclining of the planets from the parecliptic.					
♄	♃	♂	♀	☿	☾
2;30°	1;30°	1;0°	0;10°	0;45°	

third mark, it is made parallel to the alidade. And on the plate, from the third mark on the place of the edge of the ruler to an amount (being) the difference mark of that planet in the direction of the beginning of divisions in proper style a line is drawn. And this line we call the line of inclination. And for the inferior (planets) the edge of the ruler having been put along the third mark, the ruler is made to parallel the equating diameter, and from the third mark

بر صفحه در جهت مبدا اجزا به بعد رقم اختلاف ان
کوکب خط مثل بکشند پس بر نقطه استوا بعد
نقطه عرض علامتی کنند که بعد او از علامت ثانیه
بقدر بعد علامت ثالثه باشد از نقطه عرض بعیده
اگر مرکز عرض اقل از شش برج بود والا از نقطه
عرض قریبه باستعانه اجزا مسطره یا پرکار
واین علامت را بدل نقطه عرض نامیم و بر خط مثل
نقطه طلب کنیم که بعد میان آن نقطه و بدل نقطه
عرض مساوی بعد میان بدل نقطه عرض علامت
اولی باشد باستعانه پرکار یا مسطره واین را نقطه
مطلوبه خوانیم بعد ازان حرف مسطره را بنقطه
مطلوبه و بدل نقطه عرض گذرانند بنگرند و عضلو
را موازی مسطره گردانند و عضلو را موازی مسطره

f.24r

on the plate, in the direction of the beginning of divisions, to the amount of the difference mark of that
planet, the line of inclination is drawn. Then, on the equating diameter near
the point of latitude a mark is made such that its distance from the second mark shall be
to the amount of the distance of the third mark from the far latitude point
if the center of latitude is less than six signs, and otherwise from the near
latitude point with the assistance of the divisions of the ruler or compass,
and this mark we call the substitute for the point of latitude. And on the line of inclination
we seek a point such that the distance between that point and the substitute for the point of
latitude shall be equal to the distance between the substitute for the point of latitude [and]* the
first mark, with the help of a compass or ruler, and we call this the
desired point. After that, the edge of the ruler having been put along the
desired point and the substitute for the point of latitude, hold it (there), and let the alidade be made
parallel to the ruler. And, the alidade being parallel to the ruler,

*Insert , here.

وچنین نظر کنند بر مری عضاده که بر کدام جزء
افتاده است از اجزاء افق و بعد او از طرف
قطر استوا جند است این غایت مثل جزء کوکب
از قوس دایر بود از سطح مایل در سمت مین و در علویه آن
اجزاء را از غایت مثل مایل از ممثل نقصان کنند
اگر تفاضل معدله اقل از دو باشد یا اکثر از ثلثه اربع باشد
بود و الا زیاده کنند تا غایت مثل جزء کوکب
از قدر دایر از سطح بروج حاصل شود پس عضاده را
بر قطر استوا تمام کرده اینده بر حرف عضاده بر موضع
تقاطع حرف عضاده با خطی از خطوط عرض که
مساوی غایت مثل جزء کوکب از قدر دایر
از سطح مایل با بروج دلالت کند و با ان را علامت
عرض خوانیم و بعد ذلک مری عضاده را بر

f.24v

thereupon observe along (to) the pointer of the alidade (to see) on which division it has fallen of the divisions of the ring, and its distance from the direction of the equating diameter is how much. This will be the maximum [inclination]* of the part of the planet from the epicycle from the inclined surface, (i.e. deferent plane) in the inferior (planets); and in the superior (planets), those parts of the extremity of the inclination of the deferent, let (them) be decreased from the parecliptic if the adjusted anomaly is less than a quadrant or more than three quadrants, and otherwise (it) is (to be) increased until the maximum inclination of the part of the planet from the epicycle from the plane of the signs (i.e., the ecliptic) results. Then, the alidade having been erected perpendicular to the equating diameter, along the edge of the alidade, at the place of intersection of the alidade edge with (that) line of the latitude lines which is equal to the maximum inclination of the part of the planet from the epicycle to the surface which is inclined with (respect to that of the) signs, let a mark be made (there). And we call this the mark of latitude. And after that, the pointer of the alidade having been placed

*Read ميل for مثل.

مثل مرکز عرض از اجزاء حجره نهاده نگاه کنند
که علامت عرض بر کدام خط واقع شده است ان
خطوط عرض آن عرض مطلوب بود پس اگر مرکز
عرض علویه کمتر از شش برج بود عرضش شمالی باشد
واگر بیشتر باشد جنوبی بود واگر مرکز معدل
سفلیین کمتر از شش برج وخاصه معدل کمتر
از سه برج ویا بیشتر از نه برج بود ویا مرکز معدل
بیشتر از شش برج وخاصه معدل بیشتر از سه برج
وکمتر از نه برج بود عرض ثانی زهره شمالی
واز ان عطارد جنوبی بود والا از ان رأس جنوبی
وعطارد شمالی باشد اما عرض ثالث سفلیین
تقدیل ثانی کوکب را در بعد ابعد بدانند اگر مرکز
معدل کمتر از سه برج یا بیشتر از نه برج باشد

f.25r

according to the center of latitude on the divisions of the ring, note that the latitude has fallen on which line of the lines of latitude; this will be the desired latitude. Then, if the center of latitude of the superior (planet) is less than six signs its latitude will be north, and if (it) is more, (its latitude will) be south. And if the adjusted center of the inferior (planet) is less than six signs and the adjusted anomaly less than three signs or more than nine signs, or less the adjusted center more than six signs and the adjusted anomaly more than three signs and less than nine signs, the second latitude of Venus will be north, and that of Mercury south; and otherwise that of Venus south and Mercury north. But (as for) the third latitude of the inferior (planets), let the second equation of the planet be known at the greatest distance (i.e. at the apogee), if the adjusted center is less than three signs or more than nine signs,

والا تعدیل ثانی کوکب را در مقابله اوج بدانند
وثلث سدس آن تعدیل را بگیرند در زهره
وبهت عطارد تعدیل را هفت دقیقه ضرب
کنند اگر در بعد ابعد گرفته باشند والا در شش
دقیقه ضرب کنند انحراف حاصل کرده پس
مثل انحراف از خطوط عرض نگویند وعضاده را
بر خط استوا قایم گردانیده نظر کنند که خط
مطلوب یعنی که خطی که مساوی انحراف است
بر کدام جزو واقع شده است اذا جزو حرف
عضاده بران جزو علامتی کنند این علامت
عرض بود پس نود درجه بر مرکز معدل هر کوکب
زیاده کنند و حاصل را مرکز عرض خوانیم پس
مری عضاده را بر مرکز عرض نهند بار دیگر مرکز عرض

f.25v

and otherwise let the second equation of the planet be determined according to the apogee,
and take a third of a sixth of the equation for Venus,
and for Mercury, multiply the equation by seven minutes
if (it) is obtained on (the side of) the apogee, otherwise multiply it by eight
minutes; the obliquity results. Then
seek the lines of latitude according to the obliquity, and, the alidade
having been erected perpendicular to the equating line, note that the
desired line, that is, that line which is equal to the obliquity,
on which division had (it) fallen of the divisions of the edge
of the alidade. On that division make a mark. Let this be the mark
of latitude. Then let ninety degrees be increased by the adjusted center of each planet,
and the result we call the center of latitude. Then let the
pointer of the alidade be placed along the center of latitude, or opposite the center of latitude

نهند از اجزاء حجره و نظر کنند که علامت عرض
بر کدام خط واقع شد. است از خطوط عرض عرض
ثالث آن کوکب بود پس اگر مرکز معدل آن کوکب
از سه برج کمتر و یا از نه برج بیشتر باشد و خاصه
معدله از شش برج بیشتر باشد عرض ثالث
زهره جنوبی و عطارد شمالی والا زهره شمالی و
عطارد جنوبی بود اما عرض اول سفلیین مری
عضاده را بر مرکز عرض یا نظیر مرکز عرض
نهند از اجزاء حجره و نظر کنند که حرف لعضاده
با دائره عرض ان کوکب بر کدام خط از خطوط
عرض تقاطع کرده است بر آن خط علامتی کنند
و عضاده را بر قطر استوا قایم گردانیده نظر
کنند بر موضع تقاطع حرف عضاده با خط معدل

f.26r

place it, from the divisions of the ring, and observe that the latitude mark on which line of the latitude lines it has fallen, (this) will be the third latitude of that planet. Then, if the adjusted center of that planet is less than three signs, or more than nine signs, and the adjusted anomaly more than six signs, the third latitude of Venus will be south and (that of) Mercury north, and otherwise Venus will be north and Mercury south. But (as for) the first latitude of the inferior (planets), let the pointer of the alidade be put along the center of latitude, or opposite the center of latitude, from the divisions of the ring, and observe that the edge of the alidade has intersected with the latitude circle of that planet at which line of the lines of latitude. On that line let a mark be made, and, the alidade having been turned perpendicular to the equating diameter, observe the place of intersection of the edge of the alidade with the marked line,

و بران موضع از حرف عضاده علامتی کنند
این علامت عرض بود پس مری عضاده را اعنی
کنند بر مرکز عرض یا بر نظیر مرکز عرض علامت
عرض بر کدام خط که واقع است از خطوط عرض
عرض اول آن کوکب بود و عرض اول زهره
دایما شمالی و عطارد جنوبی باشد پس چون
عروض ثلثه هر کوکب را معلوم کرده باشند
اگر همه در جهت موافق باشند جمع کنند والا عرضها
موافق را جمع کرده تفاضل بین المجموع والمخالف
بگیرند عرض معدل حاصل شود و جهت عرض معدل
جهت مجموع بود در اول و جهت فضل
بود در ثانی والله اعلم باب هشتم
در معرفت ابعاد کواکب از مرکز عالم بدانکه بعد

f.26v

and at that place on the edge of the alidade make a mark. Let this be the latitude mark. Then let the pointer of the alidade be returned along(side) the center of latitude, or along the opposite to the center of latitude, the mark of latitude, on whichever line of the latitude lines (it) falls will be the first latitude of that planet. The first latitude of Venus is always north, and Mercury south. Then, since the three latitudes of each planet have been found, if all are in a similar direction, let them be added, and otherwise, like latitudes having been added, the difference between the sum and the opposing (latitude) is obtained '(that) the adjusted latitude result, and the direction of the true latitude is the direction of the sum first and the direction of the difference secondly, but God knows better. CHAPTER EIGHT On the Determination of the Distances of the Planets from the Center of the Universe. Be it known that the distance of the

علامت مرکز هر کوکب از علامت اختلاف
آن کوکب مساوی بعد مرکز جرم آن کوکب است
از مرکز عالم و بعد علامت مرکز هر کوکب از مرکز
صفحه بقدر بعد مرکز تدویر آن کوکب است
از مرکز عالم و این ابعاد معلوم است باجزائی که
نصف قطر صفحه شصت جز باشد از قبل سطر آن
وعادت اهل این صناعت بر آن جاری شده
است که تقدیر ابعاد کواکب متحیره و شمس باجزائی
می کنند که نصف قطر حوامل ایشان شصت باشد
و تقدیر ابعاد قمر باجزائی که نصف قطر مایل
شصت باشد پس اگر خواهند که از این اجزا
معلومه آن اجزا را که تقدیر بان می کنند
بدانند هر یک ازین اجزا معلومه را در مقداری ضرب

f.27r

mark of the center of each planet from the difference mark of
that planet is equal to the distance of the center of the body of that planet
from the center of the universe, and the distance of the center mark of each planet from the center of the
plate is (equal) to the amount of the distance of the epicycle center of that planet
from the center of the universe. And these distances are determined in such divisions that
half the plate diameter is sixty divisions from the sides of the two rulers.
And the custom of those who are of this art has thus become current,
that the evaluation of the distances of the planets and the sun is made with divisions
such that half the diameter(s) of their deferents shall be sixty.
And the evaluation of the distances of the moon are with divisions such that half the inclined diameter
is sixty. Then, if it is desired that from these
apparent divisions (the number of) those divisions which (are used to) evaluate them
be known, multiply each of these (values in) known divisions by a quantity

کند که نسبت آن مقدار با یک جزو از اجزاء
قطر مجوب نسبت نصف قطر صفحه بود یا نصف
قطر منطقهٔ حامل آن کوکب و ما این مقدار در هر
کوکب استخراج کرده در جدولی نهادیم و آن
جدول اینست

۱۱ ۱۱	۱۱ ۱۱	۱۱ ۱۱	۱۱ ۱۱	۱۱ ۱۱	۱۱ ۱۱	۱۱ ۱۱
۱ ۲۰	۱۱ ۲	۱۱ ۳۰	۱۱ ۱۲	۱۱ ۱۰	۱۱ ۱۰	۱۱ ۱۰

وچون نصف قطر صفحه بمقدار نصف قطر
خارج مرکز شمس است چه محیط صفحه منطقهٔ
خارج مرکز شمس است کما مر فی المقالة الاولی
و بمقدار نصف قطر مایل است ابعاد معلوم مذکور قبل
مسطر ثمن ثمان بعد بمقدار مطلوب بود والله اعلم

f.27v

such that the ratio of this quantity to one division of the divisions of the diameter be the same as the ratio of half the plate diameter to half the diameter of the deferent heaven of that planet. And we, having extracted this quantity for each planet, have put it in a table, and that table is this:

☉	☾	♄	♃	♂	♀	☿
1;0,0	1;0,0	1;9,11	1;4,55	1;19,12	1;1,3	1;13,56

And since half the plate diameter is (equal) to the amount of half the diameter of the eccentric (orbit) of the sun, thus the plate circumference is the eccentric deferent of the sun, as was prescribed in the First Treatise, and (it) is to the amount of half the diameter of the inclined (orbit). The apparent distance, ascertained by means of the two rulers, that same distance will be the desired quantity, but God knows better.

باب هفتم در معرفت رجعت و استقامت و اقامت کواکب و اوقات آن

برای معرفت رجعت و استقامت و اقامت تقویم کواکب را در ایام متوالیه حاصل کنند اگر متزاید باشد مستقیم بود و اگر متناقص بود راجع باشد و اگر نه متزاید و نه متناقص بود مقیم باشد اما معرفت مقامات رجعت و استقامت وقتی که خاصه معدله بحدود مقامات رجعت و استقامت که در جدول موضوع است برسد بعد مرکز تدویر را از مرکز عالم باجزاء نظر صفیحه بدانند و آن بعد علامت مرکز است از مرکز صفیحه و انرا بعد محفوظ خوانیم بعد ازان بعد ابعد و بعد اقرب مرکز تدویر آن کوکب را باجزاء نظر صفیحه

f.28r

CHAPTER NINE. On the Determination of Retrogradations and Forward (Motions) and Stations of the Planets and their Times.

For the determination of retrogradations, and forward motions, and stations, extract the true longitude of the planet for successive days. If it (the true longitude) is increasing it (the planet) will be (in) direct (motion), and if it is decreasing it will be retrograde, and if it is neither increasing nor decreasing it will be stationary.

But, as for the determination of direct and retrograde stations, when the adjusted anomaly reaches the limits of the retrograde and direct stations, which are placed in the table, ascertain the distance of the center of the epicycle from the center of the universe, in the divisions of the diameter of the plate.

And this is the distance of the mark of the center from the plate center, and we call this the preserved distance. After that, the greatest distance and the least distance of the epicycle center of that planet are determined in divisions of the diameter

وطریق معرفت آن چنانست که عضاد را بسطه
محاذات کند تا باوج و حضیض گذرد و بعد
نقطه تقاطع عضاد با منطقه آن کوکب از مرکز
صفحه در جانب اوج بعد ابعد بود و بجانب حضیض
بعد اقرب باشد مگر در عطارد که بعد اقرب او رابع
نیست بلکه نقطه است از منطقه رأس در تثلیث
اوج و تفاضل میان بعد ابعد و بعد اقرب کوکب
و تفاضل نهار را با جزایی که نصف قطر صفحه
شصت باشد در جدول نهادیم که عند الحاجة
الیه از بند و عمل محتاج نشوند و آن جدول اینست

f.28v

The method of determining that is thus: that the alidade is turned opposite until it passes through the apogee and perigee. The distance of the point of intersection of the alidade with the deferent of that planet from the center of the plate on the side of the apogee will be the farthest distance, and on the side of the perigee is the nearest distance, except for Mercury, for which the nearest distance is not this, but it is a point of its deferent (found by) dividing (its deferent) into three parts (from the) apogee. And the differences between the farthest distance and the nearest distance *is obtained. And the greatest and least distances* of the planets and the differences between the two in divisions such that half the diameter of the plate shall be sixty, we have put in a table that, when needed, it may be taken from there. And that table is this:

Distances	♄	♃	♂	♀	☿	
Farthest distance	55;0	58;0	50;0	60;0	56;0	Annulled
Nearest distance	49;4	52;56	40;54	57;56	45;6	
Difference	5;56	5;4	9;6	2;4	10;54	

*Marginal correction.

پس بعد محفوظ را از بعد ابعد نقصان کنند در علویه و
زهره، و بعد اقرب عطارد را از بعد محفوظ نقصان
کنند و باقی را در مابین مبداء حدود مقامات و منتهای
ضرب کنند و آن در جدول موضوع است و حاصل
ضرب را بر تفاضل بین البعدین قسمت کنند و خارج قسمت

الکواکب	زحل	مشتری	مریخ	شمس	زهره	عطارد
مبداء حدود و مقامات رجعت و نام مبداء حدود استقامت	دلو ۱۰	دلو ۱۰	دلو ۱۰	دلو ۱۰	دلو ۱۰	دلو ۱۰
منتهای حدود مقامات رجعت	حمل ۱۰	حمل ۱۰	حمل ۱۰	حمل ۱۰	حمل ۱۰	حمل ۱۰
تفاضل بینها	۱۰	۱۰	۱۰	۱۰	۱۰	۱۰

بر مبداء حدود مقامات رجعت زیاده کنند
تا مقام رجعت حاصل شود و مقام رجعت را از دور
نقصان کنند مقام استقامت باقی ماند یا خارج
قسمت را از منتهای حدود مقامات استقامت

f.29r

Then subtract the preserved distance from the farthest distance in (the case of) the superior (planets) and
Venus, and the nearest distance of Mercury, subtract it from the preserved distance.
And the remainder multiply between the beginning of the limits of the stations and their ends.
And that is placed in the table. And the
product is to be divided by the difference between the two distances, and the quotient,

The Planets	♄	♃	♂	♀	☿
Beginning of the limits of the retrograde stations and all the beginning of the limits of the direct (stations).	3°22;45°	4°7;6°	5°7;14°	5°15;45°	4°24;29°
End of the limits of the retrograde stations.	3°25;29°	4°10;11°	5°18;48°	5°18;27°	4°27;14°
The difference between the two.	2;44°	3;5°	11;34°	2;42°	2;45°

increase by the beginning of the limits of the retrograde stations,
so that the retrograde station result. And if the retrograde station is subtracted from a revolution;
the direct station will remain. Or (else) the
quotient is subtracted from the ends of the limits of the direct station

نقصان کند یا مقام استقامت حاصل شود و مقام
استقامت تا که بود در نقصان کند مقام رجعت باقی
ماند و مقام رجعت را حکم اول و مقام استقامت را
مقام ثانی خوانند پس گوید که خاصه معدله از مقام
اول بگذرد کوکب راجع شود و چون از مقام ثانی
بگذرد مستقیم شود و اگر در مقام اول بود تقسیم
رجعت خود و در مقام ثانی مقیم استقامت باشد
و مرین را غیر از استقامت نبود اما اگر کوکب مستقیم
بود و خواهیم بدانیم که کی راجع می شود خاصه معدله
را از مقام اول نقصان کنیم و باقی را بر حرکت
یکروزه خاصه قسمت کنند خارج قسمت مدتی باشد
که بعد از ان مدت راجع خواهد شدن و اگر راجع بود
و وقت استقامتش خواهند خاصه معدله را از مقام

f.29v

so that the direct station result. The direct station is subtracted from a revolution; the retrograde station will remain. And the retrograde station is called the first station; the direct station the second station. So, whenever the adjusted anomaly passes the station, the planet will become retrograde, and if it passes the second station it will become direct. And if it is in the first station its stance will be retrograde, and in the second station its stance will be direct. But the two luminaries (i.e. the sun and the moon) will never be (in) other than direct (motion). However, if a planet is direct and we want to know when it will become retrograde, the adjusted anomaly is subtracted from the (value at) first station, and the remainder is divided by the diurnal motion of the anomaly. The quotient will be such a time that after that time the retrogradation will occur. And if it is retrograde and the time until it becomes direct is required, the adjusted anomaly having been subtracted from the (value at)

ثانی نقصان کرده همان عمل کنند والله اعلم

باب دهم در معرفت نطاقات اوجی وحضیضی آفتاب و دیگر کواکب را در افلاک

خارج مرکز چهار نطاق بود مبدا نطاق اول اوج باشد ومبدا نطاق سیم حضیض واما مبدا دوم وچهارم اگر بحسب بعد کرندا نجا بود که بعد آفتاب با مرکز تدویر از مرکز عالم و مرکز خارج مرکز متساوی بود واگر بحسب سیر کرندا نجا بود که سیر آفتاب نه سریع بود و نه بطی و غیر آفتاب را از سیارات در افلاک تدویر هم چهار نطاق بود مبدا اول و سیم وزوه وحضیض مرئی و مبدا دوم وچهارم بحسب بعد انجا بود که بعد کوکب دوم کز تدویر از مرکز عالم متساوی باشد و بحسب سیر انجا بود که سیر بحسب مرکز

f.30r

second station, perform the same operation, but God knows better.

CHAPTER TEN. On the Knowledge of Apogee and Epicycle Sectors. The sun and the other planets in the eccentric orbits have four sectors. The beginning of the first sector is the apogee, and the beginning of the third sector is the perigee. But (as for) the beginning of the second sector and the fourth (sector), if it is (reckoned) on the basis of distance it will be there where the distance(s) of the sun or of the epicycle center from the center of the universe and from the eccentric center are equal. But if it is (reckoned) on the basis of movement, it will be there where the sun's movement would not be speedy and not slow. And other than the sun, of the planets, in the epicycle there are also four sectors. The beginning of the first and third will be the apparent epicyclic apogee and perigee, and the beginning of the second and fourth, according to distance will be there where the distance of the planet and the center of the epicycle from the center of the universe are equal. And according to movement (it) will be there where the movement, according to the center,

شهاباشند وان هردو باختلاف بعد مرکز تدویر از مرکز
عالم مختلف شوند و مابسا ذی نطاقات اوجی را اقرب
بعد بجهت شهرتش از دیگ اخذ کرده بر سطحه هر کوکب
علامات کردیم پس نطاقات اوجی را از قبل علامات
بدانند و اما نطاقات تدویری را چون بر صفحه علامت
مرکز داختلاف طولی کرده باشند حرف عضادها
را بر علامت مرکز نهند و نظر کنند بعلامت اختلاف
اگر بر همین کسی که مواجه عضاد باشد بحیثیتی که
علامت مرکز محاذی راس آن کس بود واقع و جنبند
اگر بعد علامت مرکز از علامت اکثر بود از بعد علامت
مرکز از مرکز صفحه کوکب در نطاق اول باشد و اگر اقل
باشد در ثانی بود و اگر علامت اختلاف طولی از
یسار آن کس باشد و بعد علامت مرکز از علامت

اختلاف

f.30v

is alone (i.e., the only one). And both of these, because of the difference in distance of the epicycle center from the center of the universe, differ. And we, having taken from the (Khāqānī) Zīj the beginnings of the apogee sectors (reckoned) according to distance, in order that they be known, we made on the deferent of every planet some marks. So the apogee sectors can be determined by means of the marks. As for the epicyclic sectors, if on the plate a mark of the center and of the longitudinal difference had been made, the edge of the alidade is put along the mark of the center, and look along to the difference mark. If it is on the right of someone who is facing the alidade under the condition that the mark of the center fall opposite the head of that person, and hence if the distance of the center mark from the [difference]* mark is larger than the distance of the center mark from the center of the plate, the planet must be in the first sector, and if it is less, it will be in the second. And if the mark of longitudinal difference is on the left of that person, and the distance of the center mark from the mark of the

*The word اختلاف is lacking in the text.

اختلاف کم از بعد علامت مرکز از مرکز صبیح بود در
نطاق ثالث بود و اگر بیشتر باشد در نطاق رابع
باشد و مرکاه که کوکب در نطاق اوّل و دوم باشد
زاید بود در عدد حساب و در نطاق سیم و چهارم
ناقص بود و حال قمر بر عکس این باشد در زیادت
و نقصان حساب و اگر کوکب در نطاق اوّل و
چهارم باشد ناقص بود در عظم و نور و اگر در دوم
و سوم باشد زاید باشد اما زیادت و نقصان نور
ماه بحسب بعد او باشد از آفتاب لکن در عظم
مانند دیگر کواکب بود و الله اعلم باب
یازدهم در معرفت خسوف ماه بر استقبال حقیقی
که در شب باشد یا در دو طرف روز واقع شود
کم از دو ساعت و چهار دقیقه گذشته از اوّل روز

f.31r

difference is less than the distance of the center mark from the center of the plate, it will be
in the third sector, and if it is more it will be in the fourth sector.
And whenever the planet is in the first and second sectors it will be
increasing in the number of the computation, and in the third and fourth sectors it will be
decreasing. But the condition of the moon is opposite this in the increase
and decrease of the computation. And if the planet is in the first or
fourth sector it will be decreased in magnitude and light, but if it is in the second
or third it must be increased. But as for the increase and lessening in light of the
moon, it will be according to its distance from the sun, but in magnitude
it is like the other planets, but God knows better. CHAPTER
ELEVEN. On the Determination of Lunar Eclipses. (At) any true opposition
which is in the night or which falls on (either of) the two sides of the day,
less than two hours and four minutes having passed from the first of the day

یا مانده از آخر روز و عرض قمر کمتر از
مقدار مجموع قطر ما و نصف قطر دایر ظل
بود خسوف ممکن بود چه اگر مقدار مجموع نصف
قطرین باشد قمر مماس دایرهٔ ظل شود و منخسف
نگردد و اگر زیاده بر مجموع نصف قطرین بود
خسوف ممکن نباشد پس اگر بعد قمر از احدی
العقدتین اکثر از دوازده درجه باشد خسوف
ممکن نخواهد بود و اگر کمتر از دوازده درجه و بیشتر
از پنجاه و نه دقیقه باشد بعضی منخسف
گردد و اگر کمتر از آن باشد خسوف کلی واقع شود
پس مرکاء که معلوم شود که خسوف واقع خواهد شد
حرف عضاده را باول حمل نهند و دقایق عرض
قمر را بدرجات رسخ کنند و بمقدار این درجات

f.31v

or remaining from the end of the day, and the moon's latitude being less than the amount of the sum of [half]* the diameter of the moon and half the diameter of the shadow circle, a lunar eclipse is possible. Otherwise, if the amount of the sum is half the two diameters the moon will be tangent to the shadow circle, and a lunar eclipse will not occur. And if it is greater than the sum of half the two diameters a lunar eclipse will not be possible. So if the distance of the moon from one of the two nodes is greater than twelve degrees a lunar eclipse will not be possible, but if it is less than twelve degrees and [the lunar latitude at conjunction]* more than twenty-nine minutes a partial lunar eclipse will occur, and if it is less than this a total lunar eclipse will befall. So, any time it become known that a lunar eclipse is to befall, put the edge of the alidade along the first of Aries and elevate the minutes of the moon's latitude into degrees, and to the amount of these

*Lacking in the text.

مرفوعه از اجزاء قطر بر صفحه علامتی کنند
و مری عضاده را حرکت دهند تا ماحد انقلابین آید
و باید که حرف عضاده بجانب علامت باشد
و حرف مسطره را برین علامت نهند بحیثیتی
که علامت خسوف که بر مسطره است برین علامت
واقع شود و راس دیگر مسطره که قرب علامت
مکث است بحرف عضاده افتد لابد از حرف
مسطره و عضاده زاویه حادث شود پس
نظر کنند که میان محدد زاویه و مرکز عضاده چند
است از اجزاء عضاده اگر باشد تضعیف کنند
پس به یک مرتبه منحط گردند یعنی هر درجه را دقیقه
شمارند آنچه حاصل شوند ساعات سقوط بود و اگر
خسوف کلی باشد آنچه بعلامت خسوف

f.32r

elevated degrees in divisions of the diameter on the plate make a mark. And the pointer of the alidade is moved until it reaches one of the two solstices, and it is necessary that the edge of the alidade be beside the mark. And the edge of the ruler is put alongside this mark in such fashion that the lunar eclipse mark, which is on the ruler, fall on this mark, and the other head of the ruler, which is near the mark of first totality, fall on the edge of the alidade. Without fail, from the edge of the ruler and the alidade an angle is formed. So observe that between the vertex of the angle and the center of the alidade how many of the divisions of the alidade there are. Whatever it is, double it, then depress it one (sexagesimal) place, i.e., count each degree a minute. That which results is the time of immersion. And if the lunar eclipse is total, that which was done with the lunar eclipse mark

کرده شد بعلامت مکث کند یا ساعات
مکث معلوم کرده پس ساعات استقبال
را در بیخ موضع نهند از اول ساعات سقوط تا
از ثانی ساعات مکث را بگاشند و ثالث
را بمجالیش بگذارند و بر رابع ساعات مکث
و بر خامس ساعات سقوط را زیاده کنند تا از اول
ساعات بدو خسوف و از ثانی بدو مکث
و از ثالث وسط خسوف و از رابع بدو انجلا
و از خامس تمام انجلا حاصل کرده و اگر خسوف
کلی بود ساعات استقبال را در پنج
موضع گذارند حاصل از اول ساعات بدو
خسوف و از ثانی وسط خسوف و از ثالث
تمام خسوف باشد بس مثل خروج عرض قمر را

f.32v

do it with the mark of (first) totality [so that]* the hours of totality can be ascertained. Then the time of opposition, put it in five places: from the first subtract the time of immersion and from the second the time of totality, and the third, put it in its (previous?) place, to the fourth add the time of totality, and to the fifth the time of immersion, so that from the first the time of beginning of the lunar eclipse, and from the second the beginning of totality, and from the third the middle of the lunar eclipse, and from the fourth the beginning of clearance, and from the fifth the end of clearance result. But if the lunar eclipse is not total the time of opposition is put in three places. The result from the first will be the time of the beginning of the lunar eclipse, and from the second the middle of the lunar eclipse, and from the third the end of the lunar eclipse. Then, according to the elevate of the lunar latitude,

*MS. has ḷ for ḳ .

از اجزاء منطره ازان راوس که بغرب علامت
مکث است بیش قسمت اصابع خسوف جوید
و ازان وجه منطره اصابع منخسفه قمر را بدانند
و الله اعلم باب دوازدهم
در معرفت کسوف آفتاب اگر اجتماع
در روز یا در یکی از دو طرف شب واقع شود
و جزء اجتماع بعد از راس و پیش از ذنب
و بعد ان جزء از عقده کمتر از شانزده درجه
باشد یا پیش از راس و بعد آن جزء از عقده کمتر
از شانزده درجه باشد یا پیش از راس
و بعد از ذنب باشد و بعد جزء از عقده کم از هفت
درجه بود کسوف ممکن باشد پس هر گاه
که کسوف ممکن باشد باز اجزاء اجتماع و ساعات

f.33r

in divisions of the ruler, from that head which is in the vicinity of the mark of totality, in front of the division, the digits are sought, and from that face of the ruler the lunar eclipse digits are ascertained, but God knows better. CHAPTER TWELVE. On the Determination of Solar Eclipses. If a conjunction befalls during the day or on one of the two sides of the night, and the part of the conjunction (is) after the head (i.e. ascending node) and before the tail (i.e. descending node), and the distance of that part from the node is less than sixteen degrees; or before the head, and after {that part from the node (is) less than sixteen degrees; or before the head and after]* the tail and the distance of the part from the node is less than seven degrees, a solar eclipse is possible. So, whenever a solar eclipse is possible, opposite the part of the conjunction and the hours of the

*Passage in braces is repeated also in the text.

f.33v

(Table of Adjusted Lunar Parallax Components)

	♒		♌		♏		♎		♍		♑		♉		
	6;57		6;48		6;16		6;0		5;[44]		5;1[2]		5;3		
	(Hours) between	Latitude Difference											(Hours) between	Latitude Difference	
7	1,41	28	1,48	20	1,53	9									7
6	1,41	27	1,47	18	1,53	9	1,51	4	1,54	6	1,48	12			6
5	1,40	26	1,45	16	1,51	8	1,50	4	1,53	7	1,48	13	1,45	22	5
P.M. 4	1,37	21	1,42	13	1,47	7	1,49	5	1,51	9	1,47	16	1,[illegible]	26	4 A.M.
3	1,28	16	1,33	9	1,36	6	1,47	7	1,47	10	1,45	20	1,30	30	3
2	1,7	11	1,11	7	1,20	6	1,34	8	1,36	15	1,30	26	1,12	35	2
1	0,37	7	0,41	6	0,50	8	1,30	13	1,16	22	1,7	31	0,40	38	1
Noon	0,0	5	0,7	8	0,15	14	0,32	20	0,13	29	0,29	37	0,40	41	Noon
1	[0]37	7	0,28	12	0,13	20	0,0	28	0,0	36	0,11	40	0,40	38	1
2	1,7	11	0,58	19	0,42	26	0,30	35	0,30	37	0,45	40	1,12	35	2
3	1,28	16	1,16	24	0,58	32	0,51	39	0,53	38	1,12	39	1,30	30	3
A.M. 4	1,37	21	1,18	29	1,8	36	1,5	40	1,8	38	1,21	36	1,46	26	4 P.M.
5	1,40	23	1,21	33	1,10	38	1,7	41	1,10	39	1,23	34	1,45	22	5
6	1,41	26	1,23	35	1,11	39	1,8	42	1,12	39	1,24	33			6
7	1,41	28	1,24	36	1,12	40									7
	♒		[♊]		[♉]		[♈]		[♐]		[♑]		♉		

(Restorations have been made by comparison with [23], f.164v, and consideration of tabular differences.)

بعد از جدول اختلاف ساعات هریک
از ساعات اختلاف و اختلاف عرض را
بگیرند و ساعات اختلاف بعد را بر ساعات
اجتماع افزایند اگر در اجتماع غربی بود و از ساعات
اجتماع بکاهند اگر شرقی باشد پس ساعت وسط
کسوف حاصل شود بعد ازان عرض قمر را
در وسط کسوف حاصل کنند اگر شمالی تفاضل
میان او و میان اختلاف و اگر جنوبی باشد
جمع کنند با عرض مرسی حاصل شود اقل از نصف
دو سه دقیقه باشد منکسف نشود و الا فلا و چون
منکسف شود بعرض مرسی و علامت کسوف
عمل کنند چنانچه بعرض قمر و علامت خسوف
کرده بودند پس ساعات بدو کسوف وسط

f.34r

distance from the table of the difference of hours (parallax), each one of the hours of difference and the difference of latitude, obtain them. And the hours of difference of distance, add (them) to the time of the conjunction if the conjunction is westerly, and subtract them from the time of the conjunction if it is easterly, so that the time of the middle of the solar eclipse result. After that, the lunar latitude at the middle of the solar eclipse is obtained; if (it is) northerly (take) the difference between it and between the difference, and if it is southerly they are added, so that the apparent latitude result. (If it is) less than thirty-three minutes a solar eclipse will occur, and otherwise not. And when it is to be eclipsed, do with the apparent latitude and the mark of the solar eclipse as had been done with the lunar latitude and the lunar eclipse mark so that the times of the beginning of the solar eclipse and the middle

وسط کسوف و انجا معلوم شود و اصابع مخسفه
بمجرد اصابع مخسفه پیدا شد و معرفت ساعات
اجتماع و استقبال در خاتمه بیاید انشاء الله تعالی
باب سیزدهم در معرفت وسط تحویل
از قبل تقویم شمس در وقت معین و ساعات
بعد تحویل مری عضاده را بر مثل تقویم شمس نهند
در وقت مفروض از اجزاء حجره و حوف مسطره
را بمرکز مستعد کرده مواز ی عضاده بیارند
و بر موضع تقاطع حرف مسطره محیط صفحه
علامتی کنند پس حرف عضاده را بر ین علامت
نهند موضع مری عضاده از اجزاء حجره وسط
تحویل بود پس استخراج وسط کنند در نصف
النهار متقدم بر وقت معروض و این وسط را

f.34v

and the middle* of the solar eclipse and clearance become known. And the solar eclipse digits are determined like the lunar eclipse digits. And the determination of the hours of conjunction and opposition, let it come in the conclusion, if God will, He is exalted!

CHAPTER THIRTEEN. On the Determination of the Mean of Transfer by Means of the True Longitude of the Sun at a Given Time, and the Hours after the Transfer. The pointer of the alidade is put according to the true longitude of the sun at the assumed time on the divisions of the ring, and the edge of the ruler having been put along the fictitious center, make it parallel to the alidade. And at the place of intersection of the edge of the ruler with the circumference of the plate let a mark be made. Then the edge of the alidade is put along this mark; the place of the pointer of the alidade in divisions of the ring will be the mean of transfer. Then the mean is extracted on the approaching noon at the assumed time, and this mean,

* وسط is repeated from the previous page.

از وسط تحویل کاهند باقی را بر حرکت یکساعته
وسط قسمت کنند خارج قسمت بعد تحویل از نصف
النهار مقدم باشد باب چهار دهم
در معرفت ارتفاع حقیقی از مری و معرفت
ارتفاع مرئی از حقیقی و اختلاف منظره
بدایره ارتفاع مری عضاده را بر اول سرطان نهند
و از مرکز بجانب اول سرطان بجهت قریب جزء
و دو دقیقه بگیرند از اجزاء عضاده و بجهت
شمس اگر آلت بزرگ باشد پیشتر کن از دو دقیقه
بگیرند و برای زهره دو دقیقه بگیرند و عند المنتهی
علامتی کنند بر صفحه سوم این را علامت منظره
خوانیم پس بر حرف عضاده برمثل بعد شمس
و با قرار از مرکز عالم در برمثل نصف بعد زهره

f.35r

let it be subtracted from the mean of transfer; the remainder is divided by the motion (in) one mean hour.
The quotient will be the distance of the transfer from the coming
noon. CHAPTER FOURTEEN.
On the Determination of the True Altitude from the Apparent and the Determination of the
Apparent Altitude from the True, and the [Parallax]*
in the Altitude Circle. The pointer of the alidade is placed at the first of Cancer,
and from the center, on the side of the first of Cancer in the case of the moon, one division
and two minutes is taken of the divisions of the alidade, and in the case of the
sun, if the instrument is large, a little more than two minutes
is taken. And for Venus two minutes is taken, and near the end
a mark is made on the plate, and we call this the mark of [parallax]*.
Then, on the edge of the alidade, according to the distance of the sun
or of the moon from the center of the universe, and according to half the distance of Venus

*MS. has مسطره ruler, for منظر.

از مرکز عالم علامت کنند و ما این را علامت
کوکب نامیم پس اگر ارتفاع مری معلوم باشد
و خواهند که ارتفاع حقیقی بدانند مری عضاده را
از اول حمل بر توالی بقدر ارتفاع مری بگردانند
و حرف بسطره را بعلامت مسطر گویند. پس از
عضاده کردانند و بر صفحه بقرب علامت کوکب
بر حرف سطر خطی کشند پس عضاده را بگردانند
تا علامت کوکب بدین خط واقع شود بعد مری
عضاده از اول حمل ارتفاع حقیقی او کوکب
باشد و تفاضل بین الارتفاعین اختلاف منظر
آن بود بدایره ارتفاع و اگر ارتفاع حقیقی
معلوم باشد و ارتفاع مری خواهند مری عضاده
را بر توالی از اول حمل بقدر ارتفاع حقیقی بگردانند

f.35v

from the center of the universe a mark is made, and we call this the (parallax) mark of the planet. Then, if the apparent altitude is known and it is desired to ascertain the true longitude, the pointer of the alidade is turned from the first of Aries along the succession (of the zodiacal signs) by the amount of the apparent altitude, and, the edge of the ruler having been put along the parallax mark, turn it parallel to the alidade, and on the plate at the planet's mark, a line is drawn alongside the edge of the ruler. Then the alidade is turned until the mark of the planet falls along this line, and the distance of the pointer of the alidade from the first of Aries is the true altitude of [that]* planet. And the difference between the two altitudes is its parallax in the circle of altitude. And if the true altitude is known and the apparent altitude is wanted, the pointer of the alidade is turned in the direction (of the signs) from the first of Aries to the amount of the true altitude.

*For او read آن.

وبر موضع علامت کوکب که بر عضاده کرده
بودند بر صفحه علامتی کنند پس حرف مسطره را
باین علامت وعلامت منظر گذرانند وعضاده را
موازی مسطره کرده انند بعد مری عضاده از اوائل
حمل ارتفاع مری آن کوکب باشد والله اعلم

باب پانزدهم در معرفت تعدیل الایام بلیالیها اهل این صناعت شبانه روزی حقیقی
را از نصف النهار گیرند تا نصف النهار تا
باختلاف بقاع مختلف نشود پس مقدار
شبانه روز حقیقی دوری باشد از معدل النهار
یا مطالع آنجا آفتاب دران شبانه روز حرکت
کند بحط استوا و حرکت آفتاب مختلف است
چه گاهی بطی السیر است وگاهی سریع السیر

f.36r

and at the place of the mark of the planet, which had been made on the alidade,
a mark is made on the plate. Then the edge of the ruler is placed
along this mark and the mark of parallax, and the alidade is turned
parallel to the ruler. The distance of the pointer of the alidade from the first of
Aries is the apparent altitude of that planet, but God knows better.
CHAPTER FIFTEEN. On the Determination of the Equation of
Time. Those having to do with this art take the true day (nychthemeron)
from noon to noon so that
in the difference of locality there will be no differing. So let the amount of the
true day be a rotation of the celestial equator (together)
with the rising (time) of that (arc through) which the sun in that day moves with respect to the (celestial)
equator. And the motion of the sun is different (i.e., variable),
since sometimes it is slow-travelling and sometimes fast-travelling,

و سر مطالع اجزاء فلک البروج متساوی نیست
پس مقادیر ایام حقیقی مختلف باشند نسبت
این هر دو تفاوت را اهل حساب تعدیل ایام
متساوی الاقدار جهت معرفت حرکات وسطی
کواکب و غیر ان پس آن زیادت بر دور را مساوی
سیر یکروز وسطا آفتاب گرفته تا ایام متعادل
متکافی باشند و این ایام متساوی را ایام
وسطی خوانند و مقادیر آن دوری باشد از معدل
النهار ما قوسی مساوی سیر یکروز وسطا آفتاب
و تفاوت میان این ایام و ایام حقیقی را تعدیل
الایام باللیالی خوانند پس بجهت معرفت تعدیل
الایام باللیالی تقویم شمس و وسطش را حاصل
کنند در وقت مفروض پس بر وسط شمس حرج

f.36v

and also the rising (times) of the divisions of the zodiac are not equal. So the amounts of the true days are different with respect to both differences, and those concerned with computation need days equal in amount for the knowledge of mean motions of planets, and for (problems) other than that. So that excess over a revolution has been taken equal to the travel in one day of the mean sun, so that the days of the year be sufficient, and these equal days are called mean days. And the amounts of (each of) these is a rotation of the celestial equator (together) with an arc equal to the travel of the mean sun in one day, and the difference between these days and the true days is called the equation of time. So, with regard to the determination of the equation of time, the true longitude of the sun and its mean is obtained at the assumed time. Then to the sun's mean three degrees

ربیع و هفت دقیقه و سی ثانیه زیادت
کنند و فضل مجموع این را بر مطالع تقویم شمس بگیرند
پس برای هر درجه از این فضل چهار دقیقه از دقایق
ساعات و برای هر ده دقیقه از دقایق فضل
یک دقیقه از ساعات و برای هر دقیقه از دقایق
فضل چهار ثانیه بگیرند مجموع دقایق و ثوانی ساعات
تعدیل ایام بلیالها بود از ایام و ساعات جنبی
نقصان کنند ایام وسطی بماند والله اعلم
خاتمه در عمل بلوح اتصالات هر یک از شهب
معدل و بعد ماضی و ساعات نصف النهار و ساعات
لیل را بدانند پس از وسط اطول بقدر ساعات
نصف النهار فراز کنند و راس سطر لیل را که
در خور اینست محاذی ساعات نصف النهار

f.37r

and fifty-seven minutes and thirty seconds is added,
and the excess of the sum of this over the (right) ascension of the sun is obtained.
Then, for each degree (of arc) of this excess four minutes of the
minutes of hours are taken, and for each ten minutes of the minutes of the excess (take)
one minute of hours, and for each minute of the minutes of the
excess four seconds are taken. The sum of the minutes and seconds of the hours will be the
equation of time. From the true days and hours
decrease them, the mean days will remain, but God knows better.
CONCLUSION. On the Operation of the Plate of Conjunctions. Each one: the daily rate,
and the past distance, and the time of noon, and the duration of
night, should be ascertained. Then extend the longer ruler to the amount of the time of
noon, and the head of the night ruler, which is
in the second trough, is put opposite the duration of the day

کنند از اجزاء جاسته لوح تا بعد راس سطر یوم
از راس سطر لیل بقدر ساعات نهار کرده و
سطر غدات را بر مثل ساعات لیل نهند از سطر
لیل پس زاید قائمه محاذی ساعات نصف النهار
یوم آیند و گردد از سطر غدات و آخر از سطر
یوم محاذی سطر لیل است حکم محو باشد بعد ازان
حرف سطر مدبر با خط را مثل مهب معدل از
اجزاء مسرات بکدرانند و انگشت یا سر قلم را بر مثل بعد
ماضی از اجزاء مسرات نهند و بر خطی که ازان جزء
خارج شده است برانند یا حرف سطر یا خط
پس خطی که از بنا با جزء آن ساعات و اجزاء سطرهای
سه گانه فرودی رود فرود آیند و نظر کنند که آن
خط بر از قطعه که در حکم محو است بر کدام دقیقه

f.37v

on the divisions of the margin of the plate so that the distance of the head of the day ruler from the head of the night ruler is made (equal to) the amount of the hours of the day. And the next-day ruler is put according to the hours of the night on the night ruler. Then the right angle will be opposite the hours of noon of the coming day on the next-day ruler. And that which of the day ruler is opposite the night ruler let it be disregarded. After that, the edge of the turning ruler or thread is placed according to the daily motion (or rate) in the divisions of travel, and the finger or pen-point is placed according to the past distance of the divisions of travel, and is run along the line extending from that division until (it reaches) the edge of the ruler or thread. Then descend along the line which runs from here into the divisions of the hours and the divisions of the three-fold rulers. And note that that line, except for the segment which is (to be) disregarded, falls along which minute

از کدام ساعات از کدام منطقه دارد است
آن ساعت اتصال از اول یوم یا اول لیل یا
از یوم آید و موقع آن خط از اجزاء حاشیه
ساعات بعدت از نصف النهار مقدم
و اگر بعد از ساعات اتصال از یوم یا لیل
یا نصف النهار معلوم باشد و
بعد مجهول بعکس
این عمل بعد را
بدانند
والله اعلم

f.38r

of which hour and which ruler.
This is the hour of conjunction from the first (hour) of the day or the first (hour) of the night
or of the coming day. And the place of that line on the divisions of the edge
is the hours of the distance from the previous noon.
And if the beginning of the hours of the conjunction from the day or the night
or noon is known, and
the distance unknown, by the reverse
of this operation the distance

may be ascertained,

but God knows better.

Finished.

C O M M E N T A R Y

References to the text and translation give the folio and line numbers of the Persian manuscript. Numbers enclosed in square brackets are references to the bibliography which begins on page 251.

1. The Introduction of the Manuscript (f. 2v:1 - 4v:4)

Folios 2v-4v are prefatory to the main body of the work, and the style and arrangement are typical of introductions to medieval Islamic scientific books. In contrast to the bald and unadorned language of the text proper these first passages are written in rhymed prose, in a florid and elaborate style. As was customary, the author chooses his figures of speech from the subject matter of his treatise, here astronomy.

The opening lines (f. 2v:1-10) are an invocation to God embellished with poetic allusions to the celestial portions of His creation. Assisted by a quotation from the Qur'ān, the author then modulates into a short (f. 2v:11 - 3r:2) panegyric on the Prophet.

Having thus paid his respects to established religion the theme is next announced - to enable the reader to solve astronomical problems without elaborate computation. The inventor on whom the anonymous author depends for his solution is named with fulsome praise, together with his two instruments.

The last section of the introduction is (f. 3v:13 - 4v:4) a dedication to the patron, the Ottoman Sultan Bayazid II (1447-1512), and a closing plea for forgiveness should the reader detect mistakes.

There follows (f. 4v:6 - 5v:7) a table of contents, after which the exposition proper begins.

2. Construction of the Disk and Ring (f. 5v:9 - 7r:1)

These two members together make up a large, 360° protractor, to be used for laying off all the angular distances needed in the subsequent operation of the instrument. The distinguishing peculiarity of this protractor, aside from its size, is the fact that its interior, the plate, is to be so constructed that it

can be rotated inside the ring, and fixed in any desired position. The manner of achieving this is shown in Figure 1, (opposite f.11r) in which only a few details are conjectural. The perforated tongue and the matching holes in the ring prescribed in the Persian text are not mentioned in the Nuzhah. Instead Kāshī prescribes (NS, p.270) that, the plate having been set in proper position with respect to the ring, it be fixed with a bit of wax. This detail is the only one in which the author of the Persian text has exhibited any originality whatever. In all other instances he simply chooses from among the possibilities suggested by Kāshī. The use of a circular set of fine, equally spaced holes is found also in Chaucer's equatorium, but not for reproducing the apsidal motion. (Cf. [42], pp.49-52.)

On our Figure 1 only four of the five circles demanded by the text have actually been drawn. A circular scale showing all five may be seen on page 48 of [42].

In these passages the language of the Nuzhah and that of the Persian version are parallel, but with some divergence. In speaking of the size of the instrument the former says (NS, p.251)

> The least that it is possible for its diameter to be is half of the great cubit (al-dhirāʿ al-kabīr), but it is better if it be two cubits of the Hāshimī cubits, or three cubits.

A cubit is the length of a forearm. The variety known as the common cubit contains about 54.0 cm., the great Hāshimī cubit about 66.5 cm., and the small Hāshimī cubit about 60.1 cm. (Cf. [13], p.55; [49], p.491), thus the ambiguity of the texts remains largely unresolved.

It is of interest to note that the same connection between size of the instrument and precision made by our author (f. 5v:13 - 6r:2) is made also by Chaucer ([42], p.18), and in almost the same terms.

3. The Jummal System of Writing Numbers

As was customary with Islamic astronomical works, the numbers in our text are represented by means of letters of the Arabic alphabet. Each letter has a numerical value found by coupling the letters, in the order of the old Semitic alphabet, with the sequence 1,2,3,...,10,20,30,...,100,200,300,... . Integers are indicated by proper combinations of letters in what is thus a non-place-value decimal system.

Fractions, however, are displayed in a place-value system with base sixty, the sexagesimal digits 0,1,2,...,59 being written as indicated above. We transcribe them as ordinary numerals, using commas to separate sexagesimal digits, and a semicolon as sexagesimal point. When arcs are involved, units of thirty degrees each may also be encountered, each one a zodiacal sign (burj, plural burūj). For these we use a superscript s. Thus either 9^{s} 23;7,0,54° or 293;7,0,54° means

$$293 + \frac{7}{60} + \frac{0}{60^2} + \frac{54}{60^3} \text{ degrees.}$$

Note that we write sexagesimals with the powers of sixty in the denominators increasing to the right; in the Arabic script they increase to the left.

To elevate (Persian raf^{c} kardan) a sexagesimal means, as we would put it, to move the sexagesimal point one place to the right. The first elevate (marfū^{c}) of 0;0,29,7. Its second elevate is 29;7. To depress (munḥaṭ gereftan) a sexagesimal

is to perform the inverse operation of elevating, i.e., to move the sexagesimal point to the left.

In a pure sexagesimal system, which we will use when convenient, both integer and fractional parts of real numbers are expressed in powers of sixty. For instance 1,0 = 60.

The reader will find additional material on the Arabic sexagesimal system in [15].

4. Planetary Apogees (f. 7r)

For the moon and the planets, deferent circles are to be drawn or engraved on the disk. The edge of the disk itself is to serve as the sun's deferent. Kāshī, like most of the other Islamic astronomers, regarded the planetary apogees as fixed with respect to each other and to the fixed stars. It was therefore necessary to impose the motion of precession on all the apsidal longitudes. This was to be accomplished by rotating the disk within the ring and setting it so that the apsidal lines had the proper longitudes for the time for which planetary positions were being determined.

The first step in the procedure was to assume a line of apsides for the sun and then to lay off the other apsidal lines with respect to it at the angular distances given in the table on f.7r. The entries of this table are rounded-off values from an equivalent table on f.128 of Kāshī's own Khāqānī Zīj [23]. Our Table 1 below has in its second column these numbers from the zīj and in the first column the longitudes of the apogees according to this work as of the year 781 Yazdigerd, the epoch of the zīj.

The numbers shown are completely secure, since positions of the apogees are also given in the zīj for years 782, 783,..., 790 Yazdigerd, and scribal errors can be restored by checking

Planet	Apogee	Apogee of Each Planet Less the Solar Apogee
♄	8^s 11;27,26°	5^s 10;28,17°
♃	6 0;15,0	2 29;15,51
♂	4 17;4,26	1 16;5,17
♀	2 20;14,28	11 19;15,19
☿	7 3;39,28	4 2;40,19
☉	3 0;59,9	----------

Table 1

one value against another.

From various sources, mostly Islamic, the editor has assembled a collection of thirty sets of planetary apogees. While many do not vary widely from this one, no other set is identical with it. It is probably taken from the Zīj-i Īlkhānī (No.6 in [29]) and may be based on observations made at the Marāghah observatory of Nasīr al-Dīn al-Tūsī.

5. Laying out the Planetary Deferents (f. 7r:10 - 8r:1)

After having located the apsidal lines, the next step is to mark the deferent centers. Each such center is on its respective apsidal line, at a distance from the plate center equal to the planet's eccentricity. A table of eccentricities is the first of the two tables on f. 7v, the units being sixtieths of the plate radius. The centers, having been marked permanently on the plate, the deferents can then be drawn, corresponding radii, expressed in the same units, being given in the second table on the same page.

It was customary to express planetary parameters in sixtieths of the deferent radius. Kāshī, however, gives them

all in sixtieths of the plate radius. He has shortened all the deferent radii in order to prevent the circles from running off the plate and to have them snugly nested, no two intersecting. In order to "norm" these parameters, that is, express them in the customary fashion, we have computed for each planet a "norming coefficient", 1,0/R, where R is the deferent radius as given in the text. The quotients make up Column 2 of Table 2 below. They are used to norm the eccentricities of the text. Side by side with the results (Column 4) are the eccentricities used in the Almagest. The reader will note that corresponding parameters are identical, or nearly so, except for Venus. The reading 1;3 is confirmed in the Nuzhah and in the Khāqānī Zīj [23]. In the former all

Planet	① Deferent Radii	② = 1,0/① Norming Coefficients	③ Eccentricities	④ = ②·③ Normed Eccentricities	⑤ Ptolemaic Eccentricities	⑥ Epicycle Radii	⑦ = ②·⑥ Normed Epicycle Radii	⑧ Ptolemaic Epicycle Radii
♄	52;2	1;9,11	2;58	3;25	3;25	5;38	6;30	6;30
♃	55;28	1;4,55	2;32	2;44	2;45	10;38	11;30	11;30
♂	45;27	1;19,12	4;33	6;0	6;0	30;32	40;18	39;30
♀	58;58	1;1,3	1;2	1;3	1;15	42;25	43;10	43;10
☿	51;23	1;13,56	4;52	6;0	6;0	18;13	22;27	22;30

Table 2

the planetary parameters placed on the instrument are attributed

to the Īlkhānī observations. A non-Ptolemaic eccentricity for Venus is not surprising. Nearby values used by al-Bīrūnī, al-Zarqālla, and Ibn al-Shāṭir are cited in [31].

6. The Deferents of the Two Luminaries (f. 7r:4 - 8r:1)

There is no need to norm Kāshī's parameters as given for either the sun or the moon. The solar deferent being the rim of the disk, its radius is already sixty. As for the moon, Ptolemy in the Almagest broke the 60-unit radius of the Hipparchian non-eccentric deferent into two parts, of lengths 10;19 and 49;41. These are the lunar parameters given on f. 7v.

Since the lunar apsidal line moves with great rapidity, the disk must be specially set each time the moon's position is to be determined, and the center of its deferent can be marked at any convenient place on the disk, say along the radius passing through the small tongue on its periphery.

The solar eccentricity prescribed, 2;6,9, differs considerably from the Ptolemaic 2;30 ([43], iii, 4). There is no doubt, however, as to the correctness of this reading. It appears also in the Nuzhah (NS, p.252), and twice in the Khāqānī Zīj ([23], ff. 95v and 157r). Doubtless this also is from the Īlkhānī observations; in his zīj Kāshī simply says that it is based on new observations.

It is close to the values used by other Islamic astronomers. Bīrūnī, in discussing the solar equation (cf. [30]) regards 2;5 as a rounded-off standard parameter. It is common to Habash al-Ḥāsib, Abū al-Wafā', and the Banī Mūsā. Al-Battānī ([3], vol.i, p.213) made it 2;4,45. Ibn al-Shāṭir uses, in effect, 2;7 (= 4;37-2;30, cf. [46], p.429).

7. The Deferent of Mercury (f. 8r:4-12)

The highly eccentric behavior of Mercury forced Ptolemy

to devise a model differing from that of the other planets. Instead of being fixed, the deferent center itself (D on Figure 3) moves on the arc of a circle with center at F and radius equal to three sixtieths of the deferent radius, DP. The equant center is at E, colinear with FC, C being the center of the universe, and CE = EF = FD. DP so moves that at all times $\theta_1 = \theta_2$. The resultant curve traced out by P and indicated by the succession of small circles on the figure is slightly oval, and has AR as an axis of symmetry. It can be regarded as a non-circular deferent for Mercury.

Kāshī approximates this curve by two circular arcs whose centers are at B and B' and whose radii are 51;23 sixtieths of the plate radius. As described in the text, B and B' are each 5;8 of these units on either side of the "turning center" (F). The latter, the center of the oval deferent, is to be 4;52 units along the line of apsides from the plate center.

In our Figure 3 the scale of the Ptolemaic locus has been so chosen that apogee A and perigee R coincide with the corresponding points on Kāshī's deferent. It will be observed that the latter is a good approximation to the Ptolemaic curve, the two loci practically coinciding for wide distances on either side of H and H'.

To obtain a norming coefficient for Mercury, we note that in Kāshī's units 56;0, the sum of the eccentricity (4;52) and the semimajor axis of the deferent (51;8), corresponds to 1,9;0 in Ptolemy's configuration. Hence the ratio of these two numbers, 1,9/56 = 1;13,56, is the desired coefficient.

8. An Oval Deferent for the Moon

In Appendix 1 of NS (p.289) Kāshī remarks that an oval

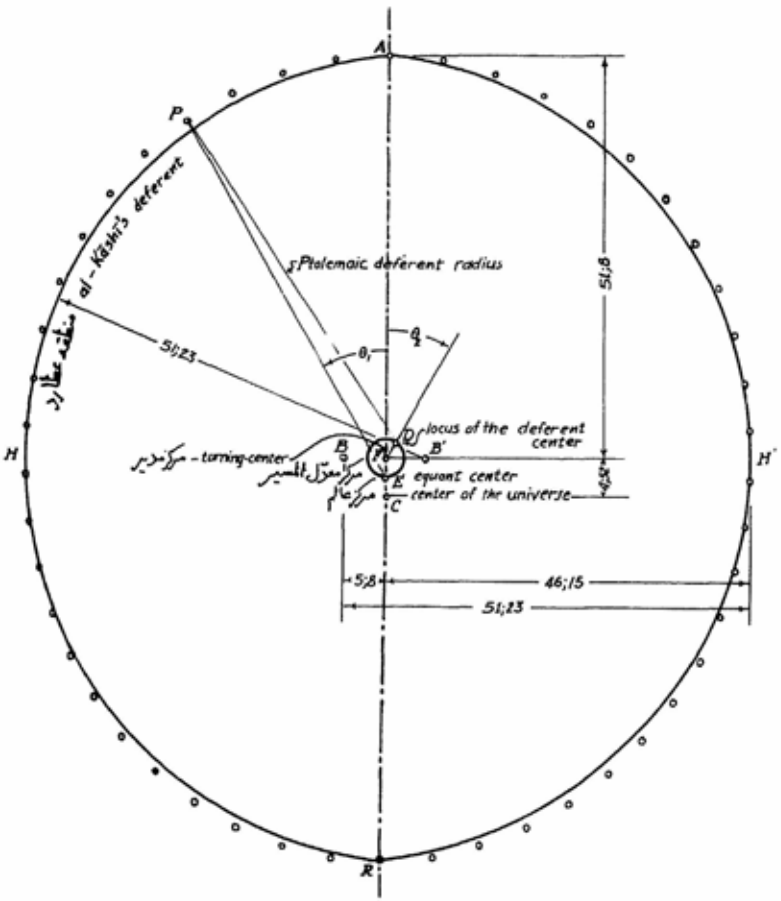

Figure 3. The Oval Deferent of Mercury

lunar deferent can be drawn permanently on the plate, thus eliminating some of the manipulations, described in Chapter II,4 of the text, for the determination of the moon's true longitude. That this is the case can easily be verified by recalling that in an abstract sense the Ptolemaic models for the moon and Mercury differ only slightly (cf. [41], Appendix I). The similarity is obscured by the rapid motion of the lunar line of apsides, but with respect to the latter the epicycle center does indeed trace out an invariant oval path in space. It is curious that whereas a number of equatorium makers laid out oval deferents for Mercury, insofar as we know only Kāshī suggested doing likewise for the moon, and then only as an afterthought.

The strongly (and erroneously) oval character of the Ptolemaic lunar orbit is portrayed in Figure 4, which shows the path of the moon plotted to scale at four day intervals through the course of a month. The direction of the mean sun on corresponding days is also indicated.

9. Marking the Equant Centers (f. 8v:1-7)

For each of the three superior planets and for Venus a point is marked on the apsidal line, outward from the deferent center by a distance equal to the eccentricity. For Mercury the equant center (E on Figure 3) is put halfway between the turning center (F), and the plate center (C). This corresponds to the Ptolemaic arrangement.

The moon has no equant, but its "opposite point" is marked on the plate, along the apsidal line, but on the opposite side of the plate center from the deferent center. The distance from the plate center to the opposite point is 10;19, the lunar eccentricity.

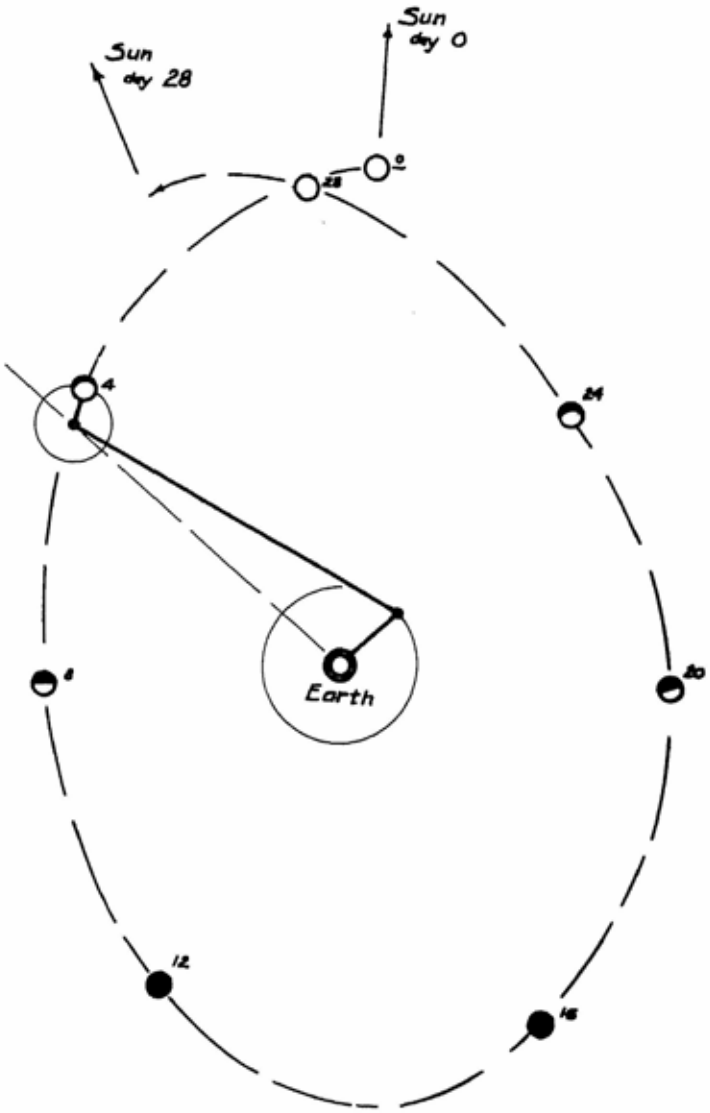

Figure 4. The Ptolemaic Lunar Orbit

10. Alternative Plate Layouts

In the Nuzhah, Kāshī does not content himself with the single arrangement of the plate and alidade given above. He describes in addition variants of the basic scheme. For instance, instead of letting the plate center represent the center of the universe for all the planets it is possible to make the deferent centers all coincide with the center of the plate. Then for each planet except Mercury the points representing the center of the universe and the equant center are placed symmetrically on the line of apsides on either side of the plate center. A plate thus laid out is said to be of the "parallel deferent" (mutawāzī al-manāṭiq) type.

Again, if no objection is made to the deferents intersecting, and if it is wished to retain the plate center as center of the universe for all planets, full advantage may be taken of the size of the plate to choose scales such that all deferents will just touch the edge of the plate. This is equivalent to demanding that the sum of the eccentricity and the deferent radius equal 1,0. Tables of the resulting parameters are given in the Nuzhah, the units being sixtieths of the plate radius.

Thirdly, in the "single deferent" (muttaḥid al-manāṭiq) type the edge of the plate itself is made to serve as the deferent for all the planets whose deferents are circular, just as was the case with the sun. For each planet this again pushes the center of the universe out along its individual apsidal line, the eccentricities and epicycle radii being the normed entries in Columns 4 and 7 of Table 2 above.

Finally, the plate may be laid out with the same apogee for all the planets, and the position of the apsidal line taken into consideration by a separate setting of the plate

within the ring for each planet. Or, since the apsidal motions are slow, the movable ring may be dispensed with altogether, the apsidal lines being laid out once and for all to a fixed position in the zodiac correct for the time when the instrument is made.

Kāshī enumerates some fifteen combinations of these various possibilities.

11. The Medieval Sine Function

The two references to sines in the text make it necessary to state that the function referred to is not the modern function, for which the radius of the defining circle is unity, but its ancestor, defined in terms of a circle of radius sixty. We distinguish between the two by writing the symbol for the medieval function with an initial capital, thus Sin θ, say. The identity relating the two functions is

$$\text{Sin } \theta = 1{,}0 \sin \theta.$$

When computations are carried out in sexagesimals, the transformation from one function to the other is trivial, since the medieval function is simply the first elevate (cf. Section 3 above) of the modern function.

In our text the plate radius is consistently taken as 1,0.

12. The Latitude Circles on the Plate (f. 8v:10 - 10r:2)

The description in f. 9r:10 - 10r:2 is not found in NK at all. It makes up part of Appendix 3 (p.291) of NS, hence represents one of Kāshī's many afterthoughts, replacing in the Persian text his original technique for solving the problem of planetary latitudes. This shows that the anonymous author

of our text worked from a copy of NS.

The two versions are practically identical in meaning. A permanent line, the equating diameter (shown on Figure 13) is to be drawn on the plate and through its center. When latitudes are being computed the plate is to be fixed in the ring so that one extremity of the equating diameter coincides with the origin on the ring, i.e. Aries 0°. Several semicircles are drawn permanently on the plate, all concentric with the ring, and all having some segment of the equating diameter as bounding diameter. The radius of the largest is to be Sin 9° (cf. Section 11 above). A convenient way of obtaining this is to join the pair of points on the ring which are both nine degrees away from the equating diameter and on the same side of it. The result is a line parallel to the equating diameter and distant from it by Sin 9°. The required semicircle can then easily be drawn, tangent to this line. Then fill the inside of this semicircle with a set of parallel lines as indicated in Figure 5 (reproduced from NS, p.292),

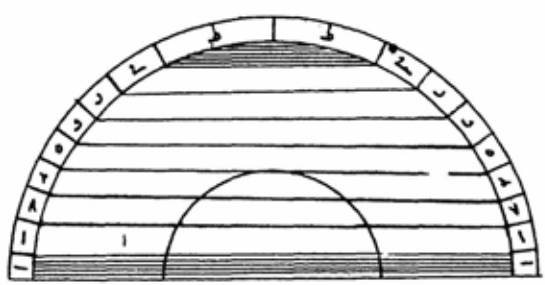

Figure 5. The Latitude Circles and Lines, from NS

these to be drawn by connecting pairs of graduations on the ring which are at equal distances from the equating diameter. The endpoints of the segments are to be marked with numbers corresponding to the degrees distance of the point-pairs from the equating diameter, 8°, 7°, 6°, and so on, and the lines at fractional distances also. These segments are known as the latitude lines. They are also shown, more crudely, on f. 11r. The basic fact to be borne in mind concerning them is that if a latitude line is marked θ its distance from the equating diameter is Sin θ.

The second semicircle, the moon's latitude circle, is to be drawn tangent to the 5° line. On Figure 5 (i.e. in the NS) it has been made erroneously tangent to the 4° line.

The last two semicircles, for Venus' first latitude and the latitude of Mercury, are drawn tangent to the ten minute and forty-five minute lines respectively.

From a practical point of view, and on a plate of any reasonable size, the Venus and Mercury semicircles would be so small as to make it impossible either to draw or to utilize them. Doubtless Kāshī was carried away by the theoretical considerations described in Section 25 below.

For material on f. 10r:2-11 see Section 33 below.

13. The Latitude Points (f. 8v:12 - 9r:9)

Eight permanent marks are to be put on the equating diameter, two for each superior planet, one each for Venus and Mercury, all on the half of the diameter opposite the first point of Aries. Distances of these points from the plate center are given in the table on f. 9r. Following is an explanation of how these particular distances were arrived at.

COMMENTARY

In the planetary theory of the Almagest the deferent planes of all the planets are slightly inclined with respect to the ecliptic, intersecting the latter along the nodal line, which passes through the center of the universe. The nodal line of each inferior planet is perpendicular to the line of (deferent) apsides of that planet. For all the other planets, however, the angles between these two lines differ. Kāshī's values are given on f. 22v. They are the same as those of the Almagest ([43], ed. of Halma, vol.ii, p.414) except for Mars, for which the Ptolemaic value is 90°. But here the divergence is apparent only, for Ptolemy computes the angle as $95\frac{1}{2}°$, subsequently deciding that use of 90° will involve only a negligible error. In effect Kāshī does the same thing.

Assume a line drawn in the deferent plane through E in Figure 12, the center of the universe, perpendicular to the nodal line MN. and intersecting the deferent in F (on the side of the apogee) and G (on the side of the perigee). Then, in the case of the superior planets, the two distances of the latitude points are EF and EG.

These distances can be computed directly in terms of the deferent radius and eccentricity. In all probability, however, Kāshī obtained them directly from the Almagest. For Saturn the corresponding distances in the Ptolemaic configuration are 1,2;10 and 57;40. For Jupiter they are 1,2;30 and 57;30. Division of each of these numbers by the proper norming coefficient obtainable from Column 2 of Table 2 gives the corresponding entries in the table on f. 9r.

As for Mars, its line of apsides is so close to FG that for two-place accuracy it is sufficient simply to add and subtract Kāshī's value of Mars' eccentricity from its deferent radius to obtain the desired quantities. Thus

$$EF = EA = ED + DA = 50;0$$

and $$GE = PE = PD - ED = 40;54.$$

For both inferior planets, maximum inclination of the epicycle plane occurs at the nodes (M and N on Figure 12). Hence the distances of the latitude points for these planets are ME and NE. But, AP being perpendicular to MN, ME = NE. And the eccentricity of Venus is so small that its deferent radius, 58;58, is used for the common distance. For Mercury, 46;0 is the distance perpendicular to the line of apsides from the center of the universe to the deferent, hence the distance to its latitude point.

14. The Alidade and Ruler (f. 11v:1 - 12r:6, 13r:4-8)

Although most of Chapter I,4 of the text is clear, there are a few obscure passages which are best clarified by first reading corresponding sections in the Nuzhah. The following is a free rendition of NS, pp.253-255, interspersed with such comments as seem appropriate. Make two rulers, says Kāshī, one like the alidade of an edged (muḥarrafah) astrolabe, but having two graduated edges, one for the true longitudes, the other for latitudes. It is as though a pair of graduated alidades were combined, back to back. Each edge shall have a semicircular projection at its midpoint, as small as practicable, and pierced by a round hole so that either edge can be pivoted to rotate about the plate center. In length the alidade should be slightly greater than the plate diameter.

The first of the two edges, called the diameter-edge (ḥarf al-quṭr), is to be graduated in equal divisions, sixtieths of the plate diameter and fractions of these to the extent possible. The divisions are to be numbered outward from the

center in both directions.

The second edge, the arcs-edge (harf al-qussī), has four sets of graduations marked upon it, each set being the projections on the alidade edge of the points of division of the ring. The four scales are obtained by effecting the projection when the alidade is in each of four different positions. For all four positions the alidade is pivoted on the plate at the hole on the arcs-edge side. One extremity of the arcs-edge is designated the head, or northern end, the other the tail or southern end. If the arc from the head of the arcs-edge to the first point of Aries on the ring is θ, then the point on the ring having a longitude of λ will project onto the edge at a distance $\cos(\lambda + \theta)$, provided that the plate radius is taken as unity. The four prescribed positions can be fixed by specifying the value of θ corresponding to each. They are

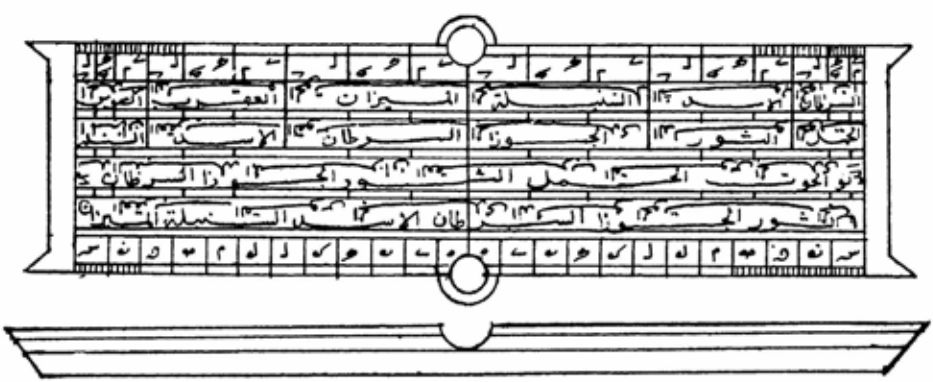

Figure 6. The Alidade and Ruler, from NS

1. $\theta = 90°$, for computing the latitudes of the moon and the second latitude of the inferior planets
2. $\theta = 0°$, for the latitude of Mars and the first and third latitudes of the inferior planets
3. $\theta = 80°$, for the latitude of Saturn
4. $\theta = 10°$, for the latitude of Jupiter.

The general appearance of the alidade is well portrayed in Figure 6, reproduced from NS, p.256. Except for the double pivot feature, it closely resembles the alidades on many existent astrolabes, for example those shown in [9], pp.121, 162, 267, and 308.

The basic principles employed in the arcs-edge, that of a linear nomogram for trigonometric functions, appears also in the dastūr quadrant (cf. [50], p.73).

The second object described in Chapter I,4, the ruler proper, is in NS specified as being narrower than the alidade, but of the same length. As indicated in Figure 6, it is to have a semicircular recess let into the center of one edge, of the same diameter as the projecting lugs on the alidades, so that the two straight-edges can be fitted snugly together. The ruler edge is to be graduated in sixtieths of the plate radius.

All three sources prescribe connecting the ruler and alidade with a chain, and the mounting of sights on the alidade to enable the user to take altitudes with the instrument. The only other equatorium into which this observational feature has been incorporated is the "Albion" developed in 1326 by Richard of Wallingford. (Cf. [42], p.128).

Descriptions of the other markings on the alidade and

ruler are deferred to the sections where the solutions of specific problems are discussed.

We are now in a position to return to the Persian text, bearing in mind that its anonymous author has here abridged the corresponding parts of the Nuzhah, sometimes so drastically as to be unintelligible. He evidently has abandoned the second set of graduations, the arcs-edge, hence he has no need of the second semicircular lug on the other side of the alidade.

As will be seen in the sequel, the main function of the ruler is to mark out directions parallel to a setting of the alidade edge made by using the graduations on the ring. According to f. 19v:9-11, the two edges are to be made parallel by seeing that the arcs of the ring intercepted between them are equal. Kāshī evidently noted the practical inconvenience of this operation, and in Appendix 4 of NS (p.297) he describes the construction of a set of parallel rulers. Our Figure 7 is reproduced from page 298 of NS.

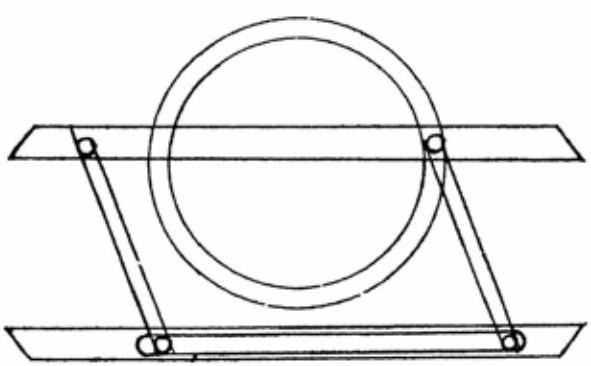

Figure 7. Parallel Rulers, from NS

15. The Difference Marks - Epicycle Radii (f. 12r:7 - 12v:3)

A set of six marks is to be engraved on one edge of the alidade at distances from the plate center specified in the table on f. 12r. Alternatively six circles may be drawn permanently on the plate, concentric with it, and having radii whose lengths are the tabular entries. In either event, each number is the length of the epicycle radius of the planet with which it is associated.

For the moon, the radius of 5;17 is close to the Ptolemaic 5;15. It is rounded off from Kāshī's own 5;16,46,36 resulting from the eclipse observations described in the opening sections of his zīj ([23] f. 4r - 6r).

Since the deferent radii of the planets proper have been shortened below the standard 60 units, their epicycle radii also have been cut down in proportion. In order to compare them with the lengths used by other astronomers it has been necessary to multiply each one by the corresponding norming coefficient found in Column 2 of Table 2. The results appear in Column 7 of the same table. It will be noted that only Mars and Mercury exhibit any difference with the Almagest values. In his zīj ([23], f. 98v) Kāshī states that the length 40;18 was determined as a consequence of the Īlkhānī observations. For Mercury the Ptolemaic 23;30 reappears on f. 110r of his zīj.

For material on f. 12v:3 - 13r:3 see Sections 36 and 40 below.

16. The Table of Mean Positions and Mean Motions (f. 13r:9 - 13v:6, 14v, 18r:9 - 19v:3)

The raw data to be fed into the instrument consist of the mean positions of the planets and their anomalies at a given

instant. These numbers are to be computed from some such table as that supplied on f. 14v of the text. The column headings of this table are self-explanatory except for the last two. The term khāṣeh-yi murakkabeh, compound anomaly, seems to be peculiar to Kāshī. He means by it the sum of the mean solar longitude and the mean anomaly of an inferior planet.

The entries in the first ten rows of the table give values of the mean longitude or anomaly for the initial instants of years 851, 852, 853,..., 860 Yazdigerd respectively. According to the marginal note on f. 18r the first day of 851 Yazdigerd (= 16 November, 1481) was chosen for epoch as having been the year of the enthronement of Bāyazīd II. The epoch of the era of Yazdigerd is 16 June, 632 A.D., the years being of just 365 days, made up of the twelve thirty-day months named in the table, plus five epagomenal days. (Cf. [10], p.298). The entries in the remaining rows display the motion of the respective longitudes or anomalies in the lengths of time indicated. These are 10, 20, 30,..., 100, 200, 300,..., 1000 Yazdigerd years, then months, days, and one hour. The numbers shown for each month are cumulative in the sense that they represent the total motion from the beginning of a year up to the beginning of the particular month. The entries opposite the numbers 1, 2, 3,..., 10 are cumulative in the same manner, for example, those in the same row as 4 give the motion for three days. The entries in the row of the second 10, however, are motions for ten days, and those in the following row are for twenty days.

All entries are in zodiacal signs, degrees and minutes, and integer multiples of twelve signs have been dropped from the reckoning. Numbers in square brackets indicate restorations to the text. The technique of checking entries is

explained in Sections 17 and 18 below. The only systematic errors in the table are those entailed by the computer having taken an erroneous 12;3,38° per year (instead of the correct 12;13,38°) for the annual mean motion of Saturn. This has affected only that part of our table which is not identical with the mean motion table of the Nuzhah.

The process of determining mean positions for a given time is explained in Chapter II,2 of the text, and requires no amplification except perhaps for the correction to be applied when a determination is to be made for a place of longitude other than that of Constantinople. Under such circumstances the difference in degrees between the longitudes of the two places is multiplied by 24/360 = 0;4 to obtain the time difference. This is added to the local time if the locality is east of Constantinople, otherwise it is subtracted.

The longitude of the latter place as given in the text, 60°, is probably rounded off from the value of 59;50° given in the Khāqānī Zīj ([23], f. 74v). Al-Battānī ([3], vol.ii, p.44) has 56;40° for the same coordinate, following Ptolemy (cf. [14], p.196). Al-Khwārizmī, however, gives 49;50°, which appears in numerous places in Ms. Vat. Gr. 1058. Hence it looks as though Kāshī's zīj value is a miscopied version of this, which our text rounds off. This still leaves us with two distinct traditions for the longitude of Constantinople.

17. "Squeezing" Mean Motion Parameters from Tables

Although the entries of our table are given to three sexagesimal places only, it is possible to "squeeze" from the tabular information a good approximation to the much more precise elements used to compute the table in the first place. The technique can best be explained by a worked example. We

deduce below the base parameter for Mercury's compound anomaly.

A first approximation to the daily motion is seen from the table to be 4;6° per day. Multiplication by 60 = 1,0 moves the sexagesimal point one place to the right to yield 4,6;0° for the distance travelled in two Persian months. The entry opposite Khurdād, however, is 8^{s} 5;33°, so we correct our first approximation to 4;5,33° for the daily motion. The product of this by 365 = 6,5 gives 24,53;45,45°. Casting off complete revolutions of 360° = 6,0°, there remains 53;45,45 ≈ 1^{s} 23;46°. The most frequent tabular difference between entries in the yearly positions is 1^{s} 23;44°, so we correct our yearly distance to 24,53;44° and multiply by 10. The result is 4,8,57;20°, the terminal digit of which is corrected to 19 by comparison with the tabular entry of 5^{s} 27;19°, and the process continues. It ends with a value of 6,54,55,31;52° for the distance travelled in 1000 Persian (i.e. Egyptian) years. Now operate backwards, successively dividing by 1000, then by 365 to obtain 24,53;43,54,43,12° and 4;5,32,41,42,27° for the basic yearly and daily rates of this parameter.

Thus the process is seen to be one of gradually building up the higher digits of the parameter by repeated multiplication, each time correcting the lower digits by comparison with the table. Each parameter has been treated in this manner to obtain the set exhibited below.

18. The Mean Motion Base Parameters

As squeezed from the table and expressed in degrees per day the mean motions used in the text are

☉	0;59,8,19,44,36°
☾	13;10,35,1,53
☾ (anomaly)	13;3,53,56,30,54

☊		0;3,10,38,18,44
☋		0;2,0,36,4,44
♃		0;4,59,16,23
♂		0;31,26,39,35,54
♀	(anomaly)	0;36,59,28,13
☿	(anomaly)	3;6,24,21,58
Apogees	(=precession)	0;0,0,8,27

The anomalistic motions shown for the two inferior planets do not result directly from the table. They are obtained by subtracting the solar motion from the compound anomaly.

Comparison of these numbers with the set of parameters independently squeezed from the Khāqānī Zīj ([23], ff. 127v - 130r) by M. Agha enables us to verify the author's statement that the positions and motions are indeed those of the zīj. The tables of the zīj, however, do not give mean longitudes as such, but the mean "centers", that is, distances of the mean planets from their respective apogees. The epoch of the zīj, moreover, is 781 Yazdigerd, and its base location is Shīrāz. The longitude of the latter place is taken as 88;0° (cf. [23], f. 73r).

We note further that in general these are the parameters of the Īlkhānī Zīj, upon which Kāshī's tables are based. For the moon, however, Kāshī has computed independent parameters based upon his observations of three lunar eclipses, those of 2 June and 26 November, 1406, and 22 May of the following year. ([23], f. 4r - 6r).

19. Mean Motions in the Nuzhah

Both versions of the Nuzhah refer to a table of mean motions. It is missing from the only extant copy of NK, but

appears on p.259 of NS. The epoch of this table is different from that of our text; it gives mean positions for Yazdigerd years 801, 802, 803,..., 810, and the base longitude is 88°, that of Shīrāz (cf. Section 18 above). Otherwise the two tables are the same.

In Appendix 5 of NS Kāshī states that he has worked out a device for eliminating even the few additions and subtractions involved in computing mean positions from a table. His invention is in essence a circular slide rule for performing algebraic addition of arcs. To construct it he prescribes drawing a family of circles on one face of the plate, concentric with it, and in sufficient number to accomodate the scales next described. These amount to a graphical mean motion table. For all planets the mean positions given in the table are marked on the circles, the origin of each circle being its intersection with the plate radius drawn to the first point of Aries on the divisions of the ring. In addition to this, arcs are laid out on suitable circles, their lengths being equal to the mean travel of the various planets and their anomalies in hundreds, and tens of years, in months, days, and fractions thereof to the extent to which marking the plate is practicable. A metal ring is made so as to fit snugly on the plate just touching the inner edge of the graduated ring surrounding the plate. Two straight rulers, of length a little greater than the plate radius, are pivoted at the plate center in such fashion that an edge of each passes through the plate center. If the date for which a planet's mean position is required happens to be the beginning of one of the years marked on a circle, simply project this mark with the edge of one of the rulers out to the graduations of the outer ring. Otherwise use the two rulers and the mean

motion scales to lay out on the inner ring an arc equal to the mean travel of the planet in question in the time elapsed between the given date and one of the years permanently marked. Then rotate the inner ring so that the initial point of the marked arc lies along the projection of the fixed point just utilized. The desired mean position will now be at the projection on the outer ring of the arc's terminal point.

Kāshī remarks that if each of the circular scales is made so that it can be individually rotated, there will be no need for the inner ring.

For material on f. 15r:1 - 17v see Section 43.

20. Determination of the Solar True Longitude (f. 19v:4 - 20r:4)

The Ptolemaic model of the sun's motion being extremely simple, the determination of its true position with the instrument is equally simple. The plate circumference itself represents the solar deferent, the apogee (represented in Figure 1 and 8 by the plate tongue) having already been fixed at its proper longitude. The fictitious center (F in either of Figures 1 or 8) stands for the center of the universe. Thus CF, the distance from the plate center to the fictitious center, is the solar eccentricity. Having computed the sun's mean longitude, lay it off as the arc AM by using the divisions of the ring. M is Kāshī's mark of the mean. Then the angle at which M is observed from F, measured from the equinoctial direction, is the true longitude. To evaluate this angle, lay the edge of the ruler along FM and rotate the alidade until it is parallel to the ruler. Then the arc AT is the required true longitude. The angular difference between the true and the mean longitudes, arc MT, is the solar equation. The length of MF, measured

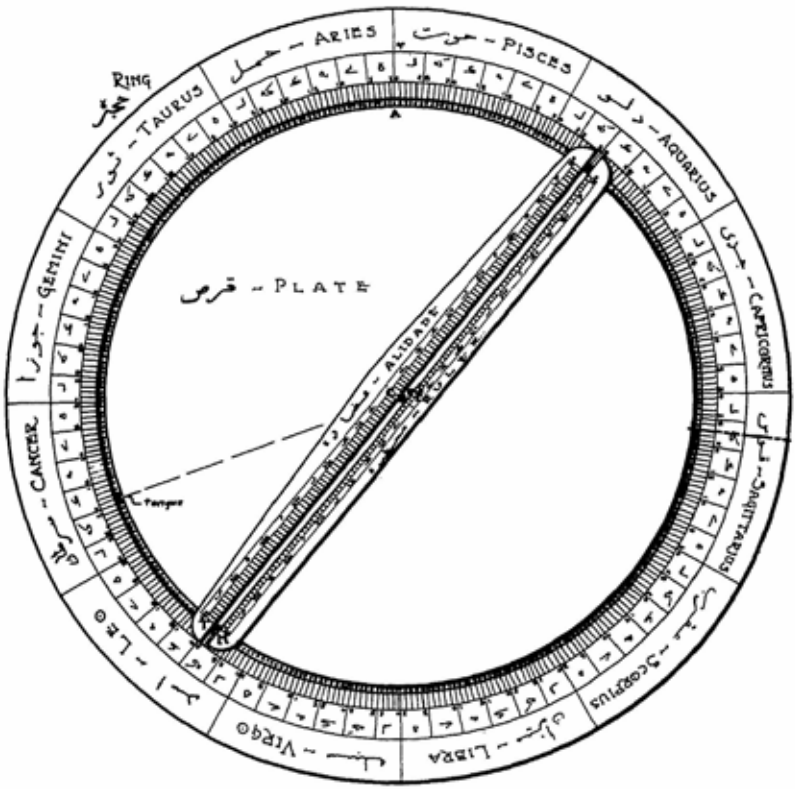

Figure 8. The Instrument, Set Up for Finding the Sun's True Longitude

with the scale marked off on the ruler, is the earth-sun distance. In this case there is no need for a norming operation since the units are the standard ones in ancient astronomy,

sixtieths of the deferent radius.

21. Determination of the Lunar True Longitude (f. 20r:5 - 20v:13)

In using the instrument to find the true ecliptic position of the moon at a given time, first compute from the mean motion table: (1) the lunar mean longitude, (2) the lunar mean anomaly, and (3) the solar mean longitude. Then mark P (see Figure 1), the position of the mean moon, on the edge of the ring. Find e, the moon's mean elongation, being the difference between (1) and (3) above. Starting from P measure clockwise along the edge of the ring an arc PS equal to 2e, the double elongation. Rotate the plate until its tongue comes opposite S and fix it here with the peg. By means of the alidade mark E the intersection of CP with the moon's deferent. E is Kāshī's mark of the center. Put the ruler along E and N, the point opposite, and rotate the alidade until it is parallel to the ruler. Mark B, the intersection of the alidade edge with the edge of the ring, the directed segments CB and NE thus being parallel and in the same sense. As the author says, it is from this point that the mean anomalistic motion is to be measured. Now rotate the alidade clockwise the amount of the anomaly, (2) above, through the arc BK. This puts the alidade in the position shown in our figure.

In order to complete the Ptolemaic lunar configuration all that remains is to lay off the epicycle radius in proper length and direction (parallel to the alidade edge) from E. Its endpoint L will then correspond to the position of the moon in space. However, this construction presents some difficulty from the practical point of view, for, as is the case with our figure, L may run off the plate entirely. Moreover, we are not primarily interested in locating L, but rather

in determining the direction of the vector CL. We recall that a lunar difference mark (Cf. f. 12r:9) has been put on the alidade edge at a distance from the center equal to the epicycle radius. The author specifically cautions us that the position of the alidade should now be such that the difference mark falls opposite the end of anomalistic motion, i.e. D being the difference mark, vector CD must have a sense opposite to vector EL. Mark the position of D on the plate and lay the ruler along DE. Since the latter equals the required vector sum of CE and EL, if the alidade is now rotated parallel to the ruler, its intersection, G, with the edge of the ring will give the required true longitude of the moon. The final passage in the chapter (f. 20v:12-13) applies only if the plate has not been set in the ring to show the proper lunar apsidal longitude at the start of the operation.

22. Determination of a Planetary True Longitude (f. 21r:1 - 21v:7)

The solution of this problem with the equatorium resembles that of the analogous lunar problem. First compute from the table of mean motions the mean longitude of the sun for the instant required, and either the planet's mean longitude (for a superior planet) or the compound anomaly (for an inferior planet). Presumably the plate has already been fixed inside the ring so that the solar and planetary apogees have their proper longitudes. Letter references in the sequel are to Figure 9, which shows the final positions of alidade and ruler in the solution of such a problem for the planet Mars. The drawing is to scale.

Rotate the alidade until its edge is at the planet's mean

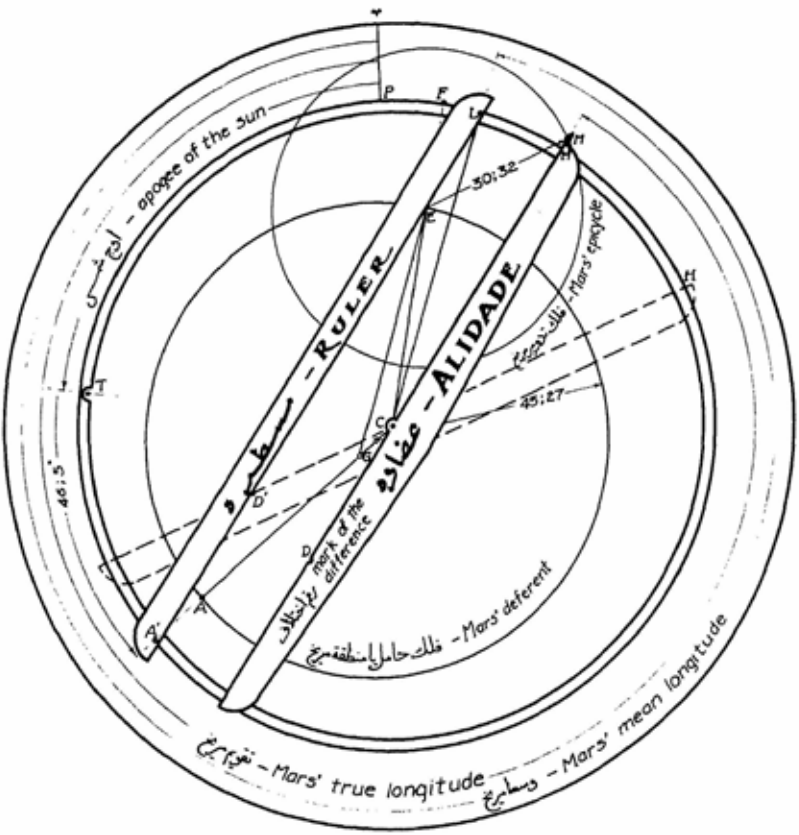

Figure 9. The Instrument, Set Up for Finding Mars' True Longitude

longitude, L, on the divisions of the ring. Place the ruler so that its edge is alongside the planet's equant-center, G,

and simultaneously parallel to the alidade edge. Where the ruler intersects the deferent, E, make the center-mark. This locates the planet's epicycle center at the given time.

For a superior planet turn the head of the alidade, that end of it opposite the side on which the difference marks (cf. f. 12r:9) have been made, until it reaches the mean solar longitude, H, and mark on the plate the point where the difference mark, D', then lies. Now put the ruler so that its edge lies along D and E, and rotate the alidade until it is parallel to the ruler. Then the intersection M of the head of the alidade with the divisions of the ring gives the required true longitude. For the vector D'C has been constructed in magnitude and direction equal to the vector EM from the epicycle center to the planet, and side CM of parallelogram CD'EM gives the direction of the vector sum of CE and D'C. And since EM is parallel to CH, EM has the required direction of the mean sun.

For the inferior planets, Venus and Mercury, the construction is of the same sort, bearing in mind that their mean longitude is the mean longitude of the sun. The same figure may be used to illustrate the configuration, although of course it will no longer be to scale. Now L is the sun's mean longitude and arc PA'H is the compound anomaly.

23. The Planetary Equations (f. 21v:8 - 22r:11)

The term equation (ta$^{\text{ʿ}}$dīl) was in general used in ancient and medieval astronomy to denote a correction, in general small, applied to a function representing some phenomenon. The usage has survived in modern astronomy in such expressions as "the equation of time".

A planet's equation in longitude was defined as the difference between its true and mean longitudes. The equation was

in turn made up of two components. The first equation, or equation of the center, is that caused by the eccentric equant; the second equation is the effect of the planet's motion on the epicycle. (Cf. [4], p.96.) Clearly the algebraic sum of the first and second equations equals the equation of the planet.

All zījes (astronomical handbooks) contained tables of both equations, essential for the computation of true longitudes. In addition, since the second equation is dependent on the first, a fairly involved interpolation scheme was necessary, to estimate the effect of the first equation on the second. It is the great advantage of an analog computer such as the equatorium that true longitudes are determined without the interposition of the equation or its components at all.

Nevertheless, should the user want to find the equations with the instrument he can do so by following the instructions in Chapter II,6. The author may have had in mind an individual who is computing a position accurately with a zīj, but who wants a quick, crude check of his partial results.

For whatever reason, having made the prescribed marks, at L, the mean position in Figure 9, and F, the projection of the epicycle center on the ring, it is clear that the arcs LF and FM are the first and second equations respectively.

When a mean position was measured from the apsidal line of the particular planet it was called the center, here arc A'HL. Addition of the first equation to the center gave the adjusted center, here arc A'HF.

The closing statement of the chapter, that if the adjusted mean longitude is subtracted from the sun's mean (for the superior planets), or from the compound anomaly (for the inferior planets), the adjusted anomaly remains, is equivalent

to the valid expression

$$PA'H = PA'F + FA'H.$$

24. The Lunar Equations (f. 22r:2)

In the ancient and medieval lunar theory the terms first equation and second equation did not denote angles analogous to those associated with the same terms in the planetary theory.

The lunar anomaly was not laid off from the epicyclic apogee of the model. Ptolemy found it necessary instead to count the anomaly from an epicyclic apogee as determined with respect to the "point opposite" (N on Figure 1), not with respect to the center of the universe, C. (Cf. [41], p.195). The first lunar equation was defined as the angular displacement in the epicyclic apogee caused by this situation. (See [23], f. 78v). On Figure 1 it is angle CEN. And since CH has been constructed parallel to EN, angle BCP also equals the first equation. But this angle, being a central angle on the plate, is measured by arc PB. P is evidently the "second mark" of our passage in the text, and B is the "mark of the beginning of the anomalistic motion", i.e. the statement in the manuscript is valid.

The second lunar equation was defined as the effect of the anomaly on the mean longitude, provided that the epicycle center were at maximum distance from the center of the universe. Since, in general, the actual distance was less than the maximum, in using the lunar tables it was necessary to add to the second equation a suitable correction. On the configuration on the instrument, however, this second equation does not appear as such, and the author of the manuscript makes no mention of it.

25. The Latitude of the Moon (f. 22r:13 - 22v:7)

The moon's orbit can be thought of as lying in a plane which makes a fixed angle of about five degrees with the plane of the ecliptic. The line of nodes, the intersection between the two planes, is not fixed, but rotates slowly in a negative direction with respect to the vernal point. Let β be the moon's latitude, λ its longitude, and λ_n the longitude of the ascending node, the ecliptic point through which the moon passes in going from southern to northern latitudes. Then we have

$$(1) \qquad \sin \beta = \sin 5^\circ \sin (\lambda - \lambda_n) = \sin 5^\circ \sin \omega,$$

an exact spherical-trigonometric relation involving ω, the argument of the latitude, the moon's ecliptic distance from the ascending node.

In computing lunar latitudes Ptolemy did not use (1), but the equivalent of the expression

$$(2) \qquad \beta = 5^\circ \sin \omega,$$

a reasonably good approximation. (Cf. [43], ed. of Halma, vol.i, p.316).

We have already described how to find λ and λ_n. Once put on the ring of the instrument, their differences can be noted directly. All versions of the text (f. 22r:13, NS p.277) say add, rather than subtract, λ and λ_n. This usage is explained by recalling that the motion of the node is contrary to the order of the zodiacal signs. Hence the nodal positions are to be plotted, as we would say, negatively, and algebraic subtraction in this case becomes addition.

Having determined ω, to complete the operation rotate the

alidade until its pointer crosses the graduations of the ring at the point corresponding to ω. Then note at which of the latitude lines the edge of the alidade crosses the moon's latitude circle described in Section 12 above. The result is the lunar latitude.

The operation is pictured in Figure 10, where we note that

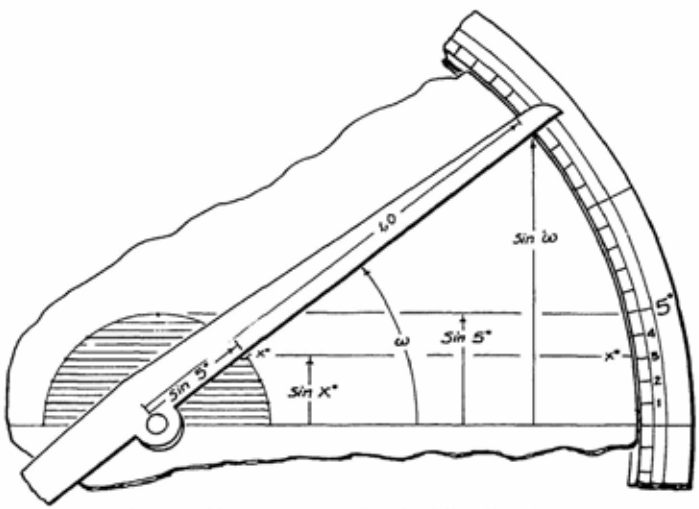

Figure 10. Use of the Latitude Lines

by similar triangles and use of the properties of the latitude lines (cf. Section 12),

$$\frac{\text{Sin } x}{\text{Sin } \omega} = \frac{\text{Sin } 5^\circ}{1{,}0} ,$$

or

$$\sin x = \sin 5^\circ \sin \omega. \tag{3}$$

Now comparison of expressions (1) and (3) shows that $x = \beta$,

that is, Kāshī's process utilizes the exact expression for determining the lunar latitude. In theory it is better than the result of using expression (2).

In the original version of the Nuzhah (NS, p.277) the method of determining the lunar latitude is as follows. Elevate the alidade so that it makes an angle of five degrees with the equating diameter. Mark on the edge of the alidade the point corresponding to ω on the lunar latitude scale. Project this point parallel to the equating diameter on to the divisions of the ring. The number on this scale corresponding to the projection is the lunar latitude. It is easy to show that this process also yields the result of applying (3).

The Chaucer equatorium manuscript [42] gives a method of determining the lunar latitude as resulting from (2). There is no provision for the computation of planetary latitudes.

26. Planetary Latitudes in the Almagest

Chapter II,7 is by far the largest section of our text, taking up about a quarter of the entire second treatise. That this is the case is not surprising, for the topic of which it treats, planetary latitudes, is the complicated result of a complicated phenomenon. In determining planetary longitudes it is convenient to regard all motion as taking place in the plane of the ecliptic, which has the effect of making the problem a two-dimensional one. This cannot be done with the motion in latitude. The accompanying theory assumes that the planes of both deferent and epicycle make small but non-zero angles with the ecliptic and with each other, some of the angles being variable.

Ptolemy regarded the latitude of a planet at any instant as being the algebraic sum of two or three component parts, known as the first latitude (β_1), second latitude (β_2) and, in the case of the inferior planets, the third latitude (β_3). In order to define these components it will be necessary for us first to introduce a number of other terms.

It was customary to designate as first diameter of the epicycle, the line of true epicyclic apsides, that is, the diameter joining the true epicyclic apogee and perigee. In Figure 11, BC, B'C', and B"C" are three positions of the first diameter. The second diameter of the epicycle is the diameter perpendicular to the first ([4], pp.61, 64). Examples from the same figure are DF, D'F' and D"F".

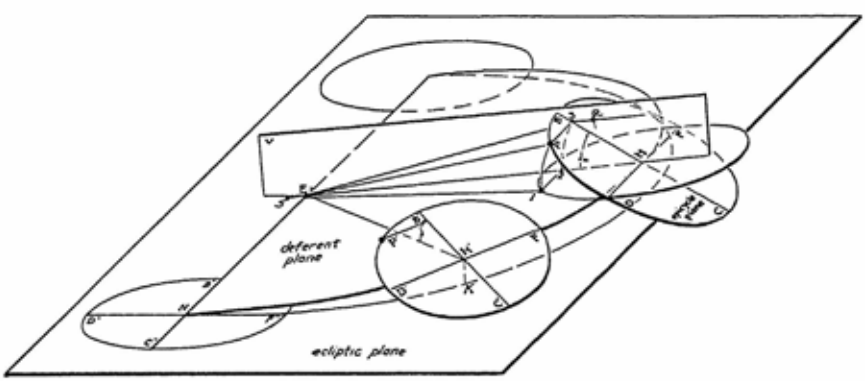

Figure 11. The Tilted Deferent and Epicycle

The first latitude is the angle made by the ecliptic plane

with the line joining the center of the universe and the epicycle center. In other words, β_1 is the portion of the total latitude due to the deviation (hereafter denoted by i) the angle at the line of nodes between the deferent and ecliptic planes.

The second latitude is the component due to the inclination (denoted by j), the tipping of the epicycle about its second diameter, the latter being maintained parallel to the ecliptic plane in the case of the superior planets.

With the two inferior planets, however, a third latitude is involved, caused by the obliquity, a tilting of the epicycle about its first diameter.

A useful concept is that of the center of latitude, denoted by $\bar{\omega}$ and defined as being the distance on the ecliptic from the ascending node of a planet to the true longitude of its epicycle center. In Figure 12 the expression

$$\bar{\omega} = \angle NEC = \angle NEA + \angle AEC$$

holds identically for all positions of C and A. Angle AEC is called the adjusted center and is denoted by $\bar{\lambda}_a$. In the case of the inferior planets, $\bar{\omega} = \bar{\lambda}_a + 90^\circ$, the line of apsides and the line of nodes being for them perpendicular. For the other planets the angles between these two lines are given in the table on f. 22v.

Chapter II,2 can be regarded as comprising four sections. The first, f. 22r:13 - 22v:7, disposes of the lunar latitude. This we have already commented on, in Section 25. The second section, f. 22v:7 - 25r:11, the longest and most involved of the four, describes simultaneously the determination of the latitudes of the superior planets and β_2 for the inferior planets. We separate the commentary on this section into two

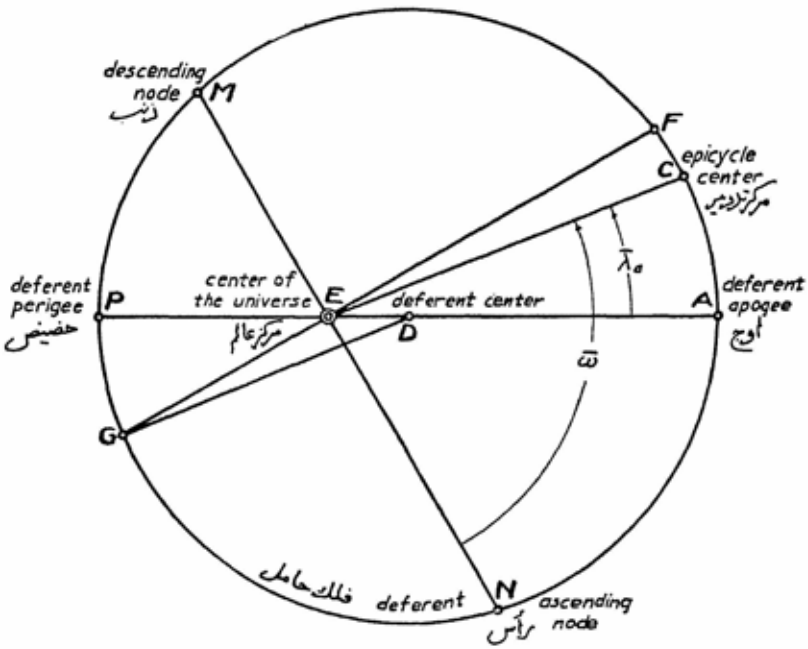

Figure 12. The Planetary Nodes

parts, Sections 28 and 30 below. The third section, f. 25r:11 - 26r:7, explains how to determine β_3 for an inferior planet and is discussed by us in Section 29. Finally f. 26r:7 - 26v:6 has to do with β_1 for an inferior planet. As being the simplest operation of the planetary latitude group, we describe it in the section immediately following this.

27. The First Latitude of the Inferior Planets (f. 26r:7 - 26v:6)

The geometric model used in the Almagest for the inferior planets has the deferent plane seesawing through a small angle north and south of the ecliptic plane about an axis in the

ecliptic plane perpendicular to the (deferent) line of apsides and passing through the center of the universe. The deviation of the deferent plane is given by the expression (cf. [8], p.199)

$$(4) \qquad i = i_m \sin \bar{\omega}, \quad i_m = \begin{cases} 0;10^\circ \text{ for Venus,} \\ -0;45 \text{ for Mercury.} \end{cases}$$

Then, if in Figure 11, H' is the location of the epicycle center at a given time, in the spherical triangle NH'K, arc H'K is β_1, the desired first latitude, angle H'KN is a right angle, and since i (angle H'NK) is small the approximate equality

$$(5) \qquad \beta_1 \approx i \sin \bar{\omega} = (i_m \sin \bar{\omega}) \sin \bar{\omega} = i_m \sin^2 \bar{\omega}$$

subsists. This is the Ptolemaic theory.

The construction described in our text yields a result which is very slightly different. The alidade is elevated from the equating diameter by an angle equal to $\bar{\omega}$. We then note at which of the latitude lines the edge of the alidade intersects the proper latitude circle (cf. Section 12). Suppose the designation of this latitude line is x. This number will then satisfy the expression

$$(6) \qquad \sin x = \sin i_m \sin \bar{\omega},$$

the transformation being of the same sort as that described for the lunar latitude in Section 25 above.

Now put the alidade perpendicular to the equating diameter and mark on its edge the point where the x latitude line crosses it. The distance from the mark to the plate center will then be Sin x. Return the alidade to its original position so that it makes an angle $\bar{\omega}$ with the equating diameter, and note the latitude line on which the mark falls. If this is the y latitude line y is the desired first latitude.

The second part of the operation is an iteration of the first with i_m replaced by x and x replaced by y. Hence we have

$$(7) \quad \sin y = \sin x \cdot \sin \bar{\omega} = (\sin i_m \cdot \sin \bar{\omega}) \sin \bar{\omega}$$
$$= \sin i_m \cdot \sin^2 \bar{\omega}.$$

Comparison of expression (5) with (7) and the small size of i_m verifies the approximate equality of the results obtained with the two expressions.

In practice the construction is completely out of the question because of the minute semicircle required to be drawn on the plate. Even from a strictly theoretical point of view, what was justified in the case of the lunar latitude cannot be defended here. In both cases a right spherical triangle was involved, the solution of which involved an expression analogous to (6). But (4) is a device for giving a simple harmonic motion, and is not an approximate equality obtained from an accurate expression such as (6).

28. The Second Latitude of the Inferior Planets (f. 22v:7 - 25r:11)

In Figure 11 three positions of a planetary epicycle are shown, its center lying at N, H, and H'. In the first position the inclination j, the angle the first diameter makes with the radius vector from the center of the universe to the epicycle center, is zero. In the second case it has taken the maximum value, j_m, and the third case illustrates a general position intermediate between the two. In all three situations the second diameter (D"F", DF, and D'F') is shown parallel to the ecliptic plane, hence β_3 and the obliquity are zero. Values

of j_m for each planet are to be found in the table on f. 23r. They are identical with those of the Almagest ([43], ed. of Halma vol.ii, p.255). In the transcription we have put a minus sign in parentheses before the entry for Venus so that the ultimate result will have the proper sign, north being taken as positive. A precise and general statement of the value of j, angle B'H'E, for all positions on the deferent, is

$$(8) \qquad j(\bar{\lambda}_a) = j_m \sin \bar{\lambda}_a = j_m \sin (\bar{\omega} - 90^\circ) = - j_m \cos \bar{\omega}$$

for the inferior planets, and

$$(9) \qquad j(\bar{\omega}) = j_m \sin \bar{\omega}$$

for the superior planets. Note that for an inferior planet maximum inclination occurs when the epicycle center is on the nodal line, in contrast to the situation pictured in Figure 11. This figure, however, shows all the elements affecting the latitude ($\beta = \beta_1 + \beta_2$) of a superior planet. For the intermediate position shown, $\bar{\omega}$ is the arc NK, the inclination, angle B'H'E, is

$$(\angle BHE) \sin \angle NEK,$$

and β is the angle P'E makes with the ecliptic plane. Also, if the relative position of the ecliptic plane there shown is ignored, the same figure may be used to illustrate the determination of the second latitude of an inferior planet.

Ptolemy regarded as prohibitively complicated a direct, general computation and tabulation of β_2 as a function of two variables, $\bar{\lambda}_a$ and α. He therefore adopted the simplification sketched below, the ensuing sacrifice of accuracy not being large. He computed β_2 for general values of α, but for $\bar{\lambda}_a = 90^\circ$,

i.e. for the position of the epicycle center at H which gives maximum inclination of the first diameter, hence maximum β_2. Here this max $\beta_2(\alpha)$ is the angle PE makes with the deferent plane. He then defines the second latitude in general as

$$\beta_2(\bar{\lambda}_a,\alpha) = \max \beta_2(\alpha) \cdot \sin \bar{\lambda}_a, \tag{10}$$

expressed in modern notation.

Kāshī finds max β_2 for a general α by means of a clever construction in the manner of descriptive geometry in which the single plane of the instrument's plate is regarded as containing the planes of the deferent, the plane denoted by v in Figure 11, and the plane of the epicycle. The equating diameter (UC in Figure 13) represents the intersection between v and the deferent plane, with H (the center of the instrument) representing the center of the epicycle. E is the latitude point (cf. Section 13) of the particular planet being dealt with, so placed that EH equals the distance (along the perpendicular to the line of nodes) from the center of the universe to the deferent of the planet. The plane v is folded down into the plane of the plate, about EH, in such fashion that its trace with the epicycle plane takes the position H3. And the plane of the epicycle is rotated into the deferent plane through an angle of j_m about its second diameter DF so that its first diameter takes the position BC.

If, now, the true length of PE (in Figure 11) can be constructed, as well as the perpendicular from P to the plane of the deferent, then the problem will be as well as solved. For if a right triangle is constructed with PE as hypotenuse and altitude equal to the perpendicular just referred to, the acute angle at its base will be the desired max β_2.

To this end the alidade is put so that its pointer crosses the ring at the graduation corresponding to the amount of the anomaly (α). The "first mark" (1 in Figure 13) is then made on the plate at the position of the permanent difference mark on the alidade edge. This makes H1 equal in length to the epicycle radius of the planet. Now make the "second mark" (2 on Figure 13), being the projection of 1 on the equating diameter.

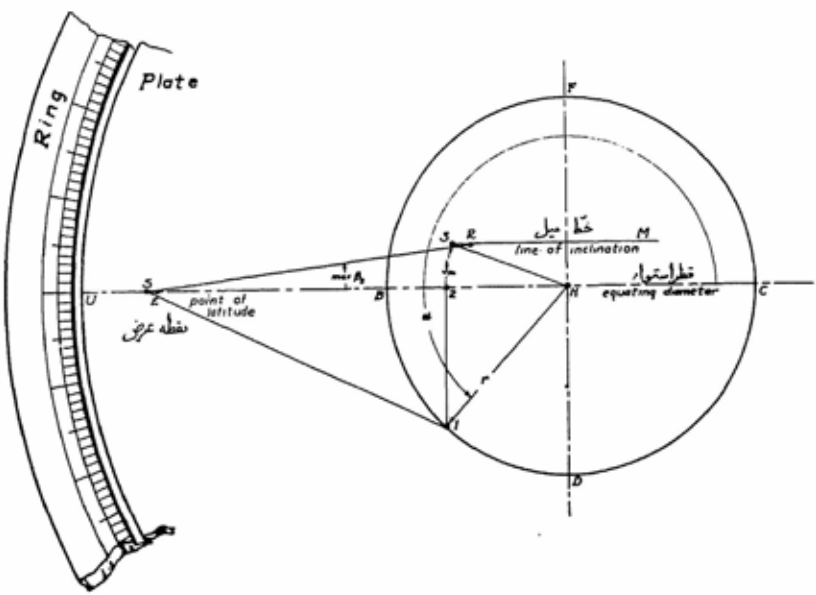

Figure 13. Construction for the Planetary Latitudes

By use of the alidade make the "third mark", 3, on the plate so that angle 3H2 = j_m and H2 = H3. Through point 3 draw a line M3 parallel to the equating diameter. The distance between this pair of parallel lines is the altitude of the desired right triangle.

To find the hypotenuse of the triangle mark point S the "substitute for the latitude point" on the equating diameter so that S2 = E3. It is as though, when the epicycle is rotated into the deferent plane, the right triangle EP3 also is flattened down into the deferent plane without change of size, the vertex E being made to slide outward along EH, 3 taking the position 2, and P the position 1. Then point S1 is the true length of EP in Figure 13.

Now, with center S and radius S1 draw an arc cutting M3 in R. Then the angle RSH is the desired max $\beta_2(\alpha)$, and it can be measured in the usual manner by placing the alidade parallel to SR and noting the point on the divisions of the ring where the pointer of the alidade falls.

The final steps in the determination consist of performing an operation analogous to the lunar latitude determination, the only differences being that the lunar latitude semicircle of radius Sin 5° is replaced by a mark on the alidade edge whose distance from the plate center is Sin max $\beta_2(\alpha)$. Thus the resulting β_2 from the instrument is given by the expression

$$(11) \qquad \sin \beta_2(\bar{\lambda}_a,\alpha) = \sin \max \beta_2(\alpha) \cdot \sin \bar{\lambda}_a,$$

which leads to a result differing slightly from that of (10). The criticism made of the β_1 determination in Section 27 above applies also to this section of the text.

The passage concludes (f. 25r:3-25r:11) with the usual complicated rules for determining the sign of the result. Negative numbers were not known to the medieval scientists. In cases where a result involved one of two directions, the result itself having issued from a combination of two or more elements, each with a direction of its own, there was nothing for it but to enumerate all possibilities in a set of rules.

The term parecliptic, occurring in this passage, requires explanation. Following Nallino ([3], vol.ii, p.352) we translate by it the word mumaththal, which is a contraction of al-falak al-mumaththal li-falak al-burūj. It is a sort of reference circle, one for each planet, lying in the plane of the ecliptic and having its center at the center of the universe. Apparently the Islamic astronomers made use of the concept because they thought of the universe in terms of a set of more or less concentric and overlapping shells, each shell containing the deferent and epicycle of a planet. On the parecliptic of each planet were projected the positions of that planet. If in any context the word ecliptic is substituted for parecliptic the essential meaning will be unchanged.

29. The Third Latitude of the Inferior Planets (f. 25r:11 - 26r:7)

In approximating the final component of the latitude Ptolemy neglects the simultaneous but small effects on it of the first two components. Thus in Figure 14, which shows the upper half (BPC) of the epicycle tilted about its first diameter (BC), the plane of EON' may be considered either as the plane of the deferent or as that of the ecliptic. E is the center of the universe, P a general position of the planet corresponding to an anomaly of α. EM is tangent to the epicycle, and the right spherical triangle OM'N' represents a portion of the celestial sphere, N' being the projection of M' on the ecliptic arc ON'. OM, which is approximately equal to q, its projection on the ecliptic, is the "second equation" of the planet. Clearly q is a function of α.

The obliquity defined in Section 26 above is measured by the spherical angle M'ON'. This angle varies sinusoidally as

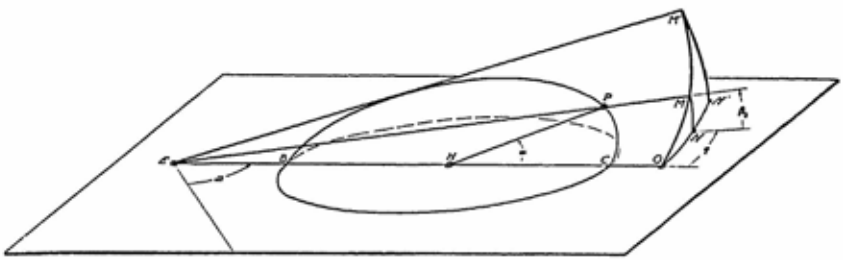

Figure 14. The Epicycle Tilted for the Third Latitude Component

H travels around the deferent, being zero when H is at the nodes and a maximum or minimum when it is in the line of apsides.

Ptolemy treats the spherical triangles MON and M'ON' as though they were plane triangles, using what is tantamount to the crudely approximate relation

$$\frac{MN}{q} = \frac{M'N'}{ON'} ,$$

or

(12) $$\beta_3 = MN \approx q\left(\frac{M'N'}{ON'}\right) = qk$$

for any fixed obliquity. The number k depends on the size of the epicycle, the deferent, the eccentricity and the maximum obliquity. Making the additional assumption that, for fixed q, β_3 varies directly as the obliquity, angle M'ON', his Almagest tables ([43], ed. of Halma, vol.ii, p.414; [3], vol.ii, p.250) are computed as though

$$(13)\quad \beta_3(\alpha,\bar{\omega}) = kq \sin \bar{\omega}, \quad k = \begin{cases} 1/18;24 & \text{for Venus} \\ \left.\begin{matrix} -0;6,8 \\ -0;7,30 \end{matrix}\right\} & \begin{matrix} \text{when } 0^\circ \le \bar{\omega} \le 180^\circ \\ \text{when } 180^\circ \le \bar{\omega} \le 360^\circ \end{matrix} \end{cases} \text{for Mercury}$$

were the general definition of the third latitude for Venus and Mercury. The opposite signs for k imply opposite tilting of the epicycle planes in the two cases. The two different values of k for Mercury are to compensate roughly for the eccentric deferent of this planet. As the epicycle leaves the ascending node it travels toward the apogee, hence, in general it is then farther from the center of the universe than after leaving the descending node. Thus the same obliquity produces less effect at the greater distance, so the value of k is smaller in absolute value on the apogee side of the line of nodes. The eccentricity of Venus is so small that the corresponding effect for it is neglected.

Except for differences in parameters, Kāshī's method is a graphical duplication of the whole scheme. The text prescribes taking a third of a sixth of Venus' second equation, i.e. 1/18. For Mercury one is to multiply the second equation by either 0;7° or 0;8° depending respectively on whether the epicycle center is or is not on the same part of the deferent as is the apogee.

It is to be noticed that in two cases Kāshī's values for k differ from the Ptolemaic ones by as much as a digit in the first sexagesimal place.

It then remains only to impose on the extreme obliquity thus found the now familiar sinusoidal transformation which employs the latitude lines on the plate. The final result is thus implicit in the expression

$$(14)\qquad \sin \beta_3(\alpha,\bar{\omega}) = \sin kq \cdot \sin \bar{\omega},$$

which is very nearly equivalent to (13).

30. Latitude of the Superior Planets (f. 22v:7 - 25r:11)

The Almagest arrangements for the superior planets are simpler on two counts than those for the inferior planets. For one thing, there is no obliquity, hence $\beta_3 = 0$. And secondly, both β_1 and β_2 vary (roughly) sinusoidally with $\bar{\omega}$, not one with $\bar{\omega}$ and the other with $\bar{\lambda}_a$ as above. In Figure 11, for instance, when the epicycle center is at N both β_1 and β_2 are zero; at H both are maximal. In the general position H', β_1 equals the great circle arc KH', which is approximately equal to $i_m \sin \bar{\omega}$. Ptolemy therefore computes as previously max $\beta_2(\alpha)$ by finding the angle EP makes with the deferent plane. Now he takes

$$\max \beta(\alpha) = \max \beta_1 + \max \beta_2(\alpha) = i_m + \max \beta_2(\alpha)$$

for general values of α. Finally he defines

$$\beta(\alpha,\bar{\omega}) = \max \beta(\alpha) \cdot \sin \bar{\omega}. \tag{15}$$

But, in contrast to the inferior planets, the deferents of the superior planets are not symmetrically placed on the line of nodes; in general, if the epicycle center is on the northern part of the deferent (i.e. $0° < \bar{\omega} < 180°$), it will be farther from the center of the universe than when it occupies a corresponding place on the southern portion ($180° < \bar{\omega} < 360°$). Hence both Ptolemy and Kāshī use two different values for EH (Figures 11 and 13) for each superior planet, depending on whether it is in the first or last two quadrants. These pairs of distances are the distances of the latitude points given in the table on f. 9r and discussed by us in Section 13. Except for this, Kāshī's construction for max $\beta_2(\alpha)$ is as explained in Section 28 above. To this add the corresponding constant i_m. Then perform the customary transformation on it involving $\sin \bar{\omega}$ to obtain the β implicit in the expression

(16) $\sin \beta(\alpha,\bar{\omega}) = \sin \max \beta(\alpha) \cdot \sin \bar{\omega}$,

which differs very slightly from (15) for the small β involved.

One passage, f. 23v:3 - 23v:11, is puzzling. It looks as though the author wants the line of inclination to make an angle of i_m with the equating diameter. This seems pointless. It is true that i_m is involved, but it should be added to the max $\beta_2(\alpha)$, angle RSH, not to the line of inclination. Also, he seems to want the line of inclination cut off in length equal to the epicycle radius. This would do no harm, but it appears unnecessary.

31. Planetary Distances (f. 26v:13 - 27v:12)

For any given time, the distance from the earth to the moon is the length of DE in Figure 1, where this figure represents the lunar configuration at the given instant. In like manner, for any planet the earth-planet distance is the length of line D'E in Figure 9. These distances having been measured in sixtieths of the plate radius, to convert them into distances measured in the standard scale, sixtieths of the respective deferent-radius, it is necessary to multiply each by a proper norming coefficient. These are given by the author in the table on f. 27v and by us in Table 2, Column 2. Determination of the solar distance has already been discussed, in Section 20 above.

32. Stations and Retrogradations (f. 28r:1 - 30r:1)

When a planet's forward motion in the zodiac ceases, it is said to be muqīm, stationary. It then becomes retrograde (rājiʿ), having passed through the first, or retrograde station (maqām-i rajʿat). After a time it again becomes stationary, passing through the second or direct station (maqām-i istiqāmat); thence

it resumes forward motion and is said to be direct (mustaqīm).

Our author devotes a chapter to the use of the Plate of Heavens in computing the time of reaching a station. He follows implicitly the directions given in the Nuzha, which is based on the theory of Ptolemy, who in turn bases his development on a proposition he attributes to Apollonius of Perga (c. 200 B.C.). In substance, this elegant theorem ([43], ed. of Halma, vol.ii, p.312) states that if the deferent center and the center of the universe coincide, then the planet (P in Figure 15) will be stationary when

$$\frac{m}{n} = \frac{v_e}{v_p} \quad ,$$

v_e being the angular velocity of the epicycle center E about O, v_p the angular velocity of P about E, and OT being perpendicular

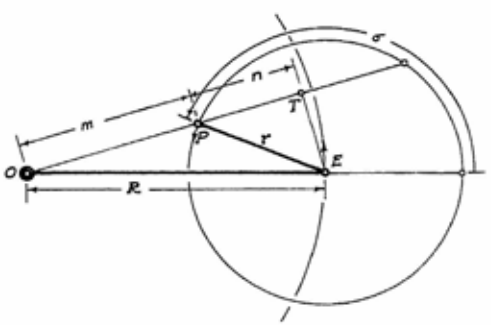

Figure 15. Diagram for Apollonius' Theorem

to ET. The station (maqām) is σ, the value of the epicyclic anomaly when the above expression obtains. Of course this is a simplification of the actual Ptolemaic model, for in the latter the center of the universe is displaced from the deferent center by a distance d. So in general OE is not a constant,

but a function of λ, here the longitude of the epicycle center measured from the deferent apogee. We call this variable radius $\rho(\lambda)$, and note that $\rho(0°) = R + d$ and $\rho(180°) = R - d$. Moreover, Apollonius' theorem applies only approximately to this model, and the location of the station on the epicycle is also a function of λ, $\sigma(\lambda)$ say. Ptolemy computes directly only three values of this function for each planet: $\sigma(0°)$, $\sigma(180°)$, and $\sigma(v_d) = \sigma$, now the location of the station when the epicycle center is at the mean distance. For intermediate values of the argument he uses what amounts to the interpolation scheme

$$(17)\qquad \sigma(\lambda) = \begin{cases} \sigma + \dfrac{[\rho(\lambda) - R][\sigma(0°) - \sigma]}{d}, & 0 \le \lambda \le v_d, \\ \sigma + \dfrac{[\rho(\lambda) - R][\sigma - \sigma(180°)]}{d}, & v_d \le \lambda \le 180°. \end{cases}$$

Thus he makes the change in $\sigma(\lambda)$ proportional to the change in $\rho(\lambda)$ for corresponding λ. (Cf. [3], vol.ii, p.246).

The arrangements in our manuscript are somewhat different, at least in appearance. On f. 29r is a table of $\sigma(0°)$ and $\sigma(180°)$, reproduced below together with the corresponding values determined by Ptolemy ([43], ed. of Halma, vol.ii, p.355), al-Battānī ([3], vol.ii, p.138), and Ulugh Beg [54]. It should be remembered that these numbers depend on four parameters, not only on d and r, but also on the mean motions and mean anomalistic rates of the planets involved. Kāshī makes no mention or use of an independently computed $\sigma(v_d)$; instead he puts

$$(18)\qquad \sigma(\lambda) = \sigma(0°) + \frac{[\rho(0°)-\rho(\lambda)][\sigma(180°)-\sigma(0°)]}{2d}.$$

The difference between this and expression (17) is more apparent than real. In fact, if $\sigma(v_d) = [\sigma(180°)-\sigma(0°)]/2$, they are equivalent, and Ptolemy's values for $\sigma(v_d)$ are practically the mean between his extreme values.

	σ(0°)				σ(180°)			
	Almagest	Kāshī	Ulugh Beg	al-Battānī	Almagest	Kāshī	Ulugh Beg	al-Battānī
♄	3°22;45°	3°22;45°	3°22;10°	3°22;45°	3°25;29°	3°25;29°	3°24;49°	3°25;29°
♃	4° 4;5°	4° 7;6°	4° 3;40°	4° 4;5°	4° 7;11°	4°10;11°	4° 6;51°	4° 7;11°
♂	5° 7;28°	5° 7;14°	5° 5;36°	5° 7;33°	5°19;9°	5°18;48°	5°19;42°	5°19;14°
♀	5°15;51°	5°15;45°	5°16;12°	5°15;53°	5°18;21°	5°18;27°	5°18;4°	5°18;21°
☿	4°27;14°	4°24;29°	4°27;14°	4°27;13°	4°24;40°	4°27;14°	4°24;40°	4°24;40°

Table 3

The values laid out on the plate for ρ(0°), ρ(180°) and 2d are displayed on f. 28v for the convenience of the user. ρ(λ), the "preserved distance", is to be obtained by direct measurement with the ruler.

The first and second stations, being symmetrically disposed on the epicycle with respect to the true (epicyclic) apogee, computation of the one for a given λ gives the other immediately.

The final step in the determination consists simply of evaluating (σ(λ)-α)/ȧ where α is the anomaly for the time being and ȧ is the rate of change of α with respect to time. The result will be the time until station is reached, in the same units used to express ȧ.

Special remarks concerning Mercury (f. 28v:5, 29r:2) are occasioned by the oval deferent of this planet (see Figure 3), the effect of which is to bring the epicycle nearest the center of the universe at two different points, not at the deferent perigee as is the case with all other planets. These two points are about 120° away from the deferent apogee, as is inferred

from Kāshī's rule, and as can be verified by inspection of the last column in the Almagest table of Mercury's equations ([43], ed. of Halma, vol.ii, p.309).

33. The Planetary Sectors (f. 10r:2-11, 10v, 30r:2 - 31r:10)

The subject of Chapter II,10 in our text is treated in [28], which may be consulted for details by the reader. We content ourselves here with a minimum for understanding of the text, beginning with a restatement of the definitions with which the chapter begins.

For certain astrological purposes, it was customary to divide the deferent and the epicycle into four sectors each, called by our author apogee sectors (nitāqāt-i aujī) and epicyclic sectors (nitāqāt-i tadvīrī) respectively. There were two categories of sectors, those computed according to distance, and those according to velocity (harakah), or, as our author puts it, movement (sayr).

In all cases the beginning of the first sector was at the (deferent or epicyclic) apogee, the beginning of the third sector at the perigee.

The beginnings of the second and fourth distance sectors on either the deferent or the epicycle were defined as those points at which an object moving on the circle in question is at its mean distance from the center of the universe.

For the velocity sectors, the beginnings of the second and fourth are those positions of a point moving on the deferent or epicycle at which its angular velocity, as viewed from the center of the universe, attains its mean value.

The endpoint of each first sector coincides with the beginning of the second, and so on.

The table on f. 10v locates the sector boundaries for all

categories. The entries have evidently been obtained from Kāshī's zīj ([23], f. 141v), uniformly rounded off to minutes. We next address ourselves to the question of how these values were determined.

The problem is simplest for the sun, which has no epicycle, hence no epicyclic sectors, and no equant. If d and R are the deferent eccentricity and radius respectively, then the arc of the deferent from the beginning of the first sector to the beginning of the second, that is, the amplitude of the first solar distance sector is

$$90^\circ + \arcsin \frac{d}{2R} . \tag{19}$$

This disposes of all other solar sectors of this category, for, as always, the sectors are symmetrically located with respect to the line of apsides. Hence the amplitudes of the first and fourth sectors are always equal, while the first and second are supplementary.

The amplitude of the first solar velocity sector is

$$90^\circ + \arcsin \frac{d}{R} . \tag{20}$$

With the planets, the location of the initial point of the second deferent distance sector can also be obtained from (19) above. For use in the zījes, however, it was convenient to tabulate the angle subtended by the first sector at the equant rather than at the deferent center. For small d a good approximation to the former angle is

$$90^\circ + \arcsin \frac{3d}{2R} . \tag{21}$$

For a planetary deferent velocity sector the angle subtended by the first sector at the equant is approximately that given

by expression (20) above.

For the tabulated values of the epicycle distance and velocity sectors the expressions

(22) $$90^\circ + \arcsin \frac{r}{2(R+d)} ,$$

and

(23) $$90^\circ + \arcsin \frac{r}{R+d}$$

respectively have evidently been used by the original computer of the zīj, where r is the epicycle radius.

In the zīj itself the "equation" also is tabulated, that is, the variation in the results when R+d is replaced by R-d. Something of the sort is necessary to take account of the variation in the distance from earth to epicycle center, a variation which causes small changes in the size of the sectors. For intermediate positions between the extremes of R+d and R-d an interpolation scheme was used which is not mentioned in our text. It is to this variation, however, which the author has reference in his statement on f. 30v:1.

Independent computations using Kāshī's parameters in the expressions above have resulted in a verification of all the entries in the text's table of sectors except for some involving Mercury and the moon, which, having special models, demand special treatment.

For Mercury's deferent distance sectors, the last column in the Almagest table of Mercury's equations ([43], ed. of Halma, vol.ii, p.309) measures the variation in the maximum size of the equation due to the epicycle as compared with its mean value. Hence any point at which this function vanishes marks a position at which the epicycle is at mean distance, i.e. the initial point of the second deferent distance sector.

Interpolation in this table yields a zero when the argument is 67;13°, a result reasonably close to our tabular value of 67;44°.

As for the deferent velocity sectors of Mercury, (19) above is applied as though it were valid for a non-circular as well as a circular deferent. Expressions (22) and (23) are used for the epicycle sectors of both Mercury and the moon.

Kāshī's definition of the initial point of the second lunar deferent distance sector is evidently taken to be the value of the double elongation at which the epicycle center will be at mean distance from the center of the universe. This is

$$\text{arc cos } \frac{d}{2R} = \text{arc cos } \left(\frac{10;19}{2(49;41)}\right) = 84;2°,$$

as required by the table.

In the Almagest table of lunar equations ([43], ed. of Halma, vol.i, p.316) the value of the double elongation which gives maximum displacement of the epicyclic apogee (Column 3 in the Almagest table) is 114°. This is our tabular entry for the moon's deferent velocity sector. In fact, however, the lunar epicycle travels on the deferent in such fashion that its angular velocity as viewed from the center of the universe is a constant. Hence the concept of deferent velocity sectors as defined above has no meaning. In some way Kāshī must have associated the 114° with the conditions which identify the mean angular velocity of the epicycle centers of the planets, but how he did so is not clear.

So much for the table. As for the instrument, since all deferents are marked on the plate, it is an easy matter to mark also the boundaries of the deferent sectors, as prescribed in the text (ff. 10r:2, 30v:2).

The epicycles as such do not appear on the instrument,

and to determine the epicycle sector of a planet at a given time recourse must be had to manipulations with the instrument. The author's directions in f. 30v:5 - 31r:2 have reference to a person standing at the periphery of the horizontally placed instrument, opposite the planet's epicycle center, and looking toward the center of the instrument. If the position of the planet is on his right this implies that the true longitude exceeds the adjusted mean longitude (i.e. the longitude of the epicycle center) and that the planet is in either the first or second epicyclic sector. If it is on the left it is in the third or fourth. Compare the distances of the planet and the epicycle center from the center of the universe. If the first exceeds the second the planet is in either the first or fourth epicyclic distance sector. If the reverse is the case it is in either the second or third. The combination of these two criteria suffices to locate the planet's epicyclic distance sector. The author gives no directions for the velocity sectors.

The terms "increasing" (or "increased") and "decreasing" (or "decreased") as used in the last part of this chapter (f. 31r:3-9) are explained by Bīrūnī in [4](p.203). "Increasing in computation" means that the equation is positive. The author is here referring to the epicycle sectors only. "Increased in magnitude" refers to the celestial object's apparent size, which varies inversely with its distance. When it is in the second or third epicyclic sector its apparent size will be increased over its mean value, hence the term.

34. Prediction of Lunar Eclipses (f. 31r:11 - 33r:3)

A lunar eclipse occurs whenever the moon enters the shadow cast by the earth. If for any time during the eclipse the moon

is completely immersed in the shadow the eclipse is said to be total, otherwise it is partial. Our author seeks to describe eclipses to the extent of determining the times of (a) first contact of moon and shadow, (b) first totality, if the eclipse is total, (c) the middle of the eclipse, (d) the beginning of clearance, i.e. the end of totality, and (e) complete clearance. For a partial eclipse the magnitude also is desired.

Since the depth and duration of an eclipse is determined by how closely the broken line sun-earth-moon approaches straightness, it is clear that a necessary condition for a lunar eclipse is that sun and moon be in opposition, i.e. have longitudes differing by 180°. The condition is not sufficient, however, for the moon does not travel on the ecliptic, but along a second great circle which intersects the ecliptic at an angle of about five degrees. This circle rotates slowly westward so that the nodes, the points of intersection between the two circles, have a motion of about nineteen degrees per year. The moon's distance from the ecliptic, its latitude, is the element which determines whether or not an eclipse will occur at a given opposition.

Our author makes several simplifying assumptions. For one thing he regards the earth-moon distance as a constant, hence the apparent size of the lunar disk as viewed from the earth is also constant. The same sort of assumption for the earth-sun distance implies that the width of the shadow cone where it is cut by the moon is likewise a constant. These simplifications have the effect of making an essentially three-dimensional problem two dimensional, for now we can confine our attention to the surface of the sphere, concentric with the earth, in which the moon's center moves. As a final simplification, let Figure 16 represent to some scale a small

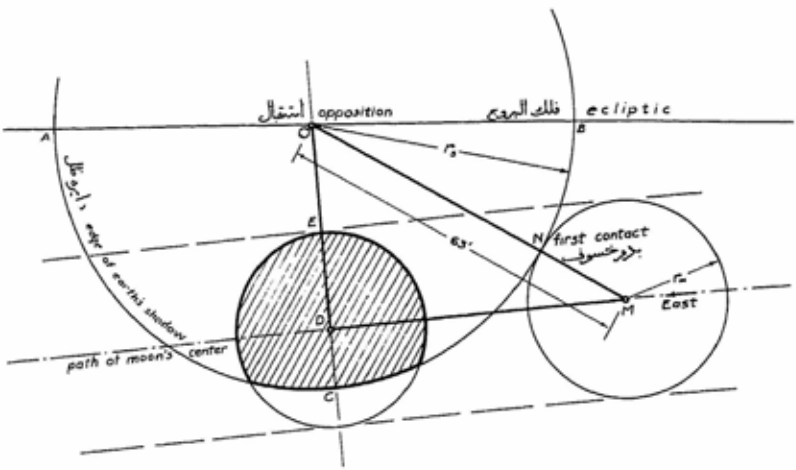

Figure 16. A Diagram for a Partial Lunar Eclipse

portion of this spherical surface, with O marking a point on the ecliptic AB at which moon and sun are in opposition. The path of the moon's center is MD and its apparent radius is MN. The intersection of the earth's shadow and the sphere at the time of opposition is represented by the circle ABC. OD is perpendicular to DM, and since the latter makes with the ecliptic an angle not greater than five degrees, the length of OD is a fair approximation to the moon's latitude at the time of conjunction. If the moon is tangent to the shadow circle at N, then, in the right triangle ODM, OM is the sum of the moon's apparent radius and the shadow radius, and DM is an approximation to the difference between the moon's elongation at first contact and at opposition. DM cannot be regarded as the difference in lunar longitudes between these

times, for in the same period the sun itself, hence the earth's shadow, will have made some progress along the ecliptic.

When an eclipse is partial, as in the one illustrated in Figure 16, its magnitude is the length of EC measured in (eclipse) digits, twelfths of the apparent lunar diameter. Figure 17, on the other hand, shows a total eclipse. Here

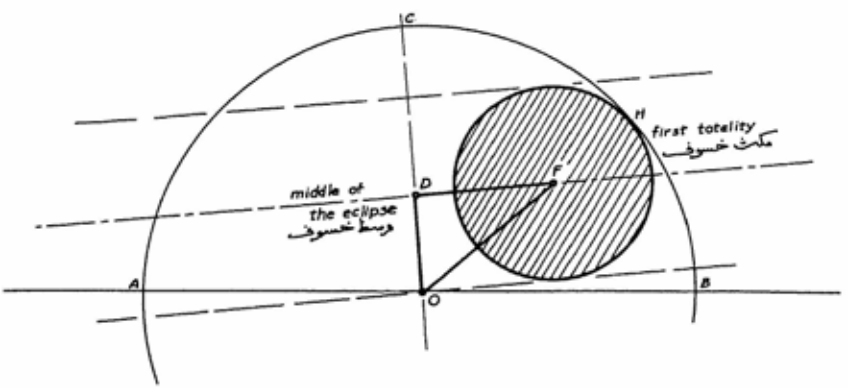

Figure 17. A Diagram for a Total Lunar Eclipse

the beginning of totality is of interest, and this occurs when the moon is tangent internally to the shadow circle, as at H. In both cases the approximate middle of the eclipse is marked by the arrival of the moon's center at D, this corresponding to the time of opposition. The ends of totality and of contact are located on the eastward side of OC symmetrically with their beginnings.

35. Lunar Eclipse Limits

With this background we now examine the text of Chapter

II,11, beginning with f. 31v:1, a statement to the effect that a necessary and sufficient condition that an eclipse occur is that the lunar latitude at opposition (β) be less than the sum of the apparent radii of moon (r_m) and shadow (r_s). The next sentence, f. 31v:5, looks corrupt, and comparison with the corresponding passage in NS (p.280) confirms our suspicion. It says:

> If the lunar latitude at the conjunction is more than sixty-three minutes, (then) undoubtedly its distance from the node will be more than twelve degrees, and so there will be no eclipse. But if it is less than that and greater than twenty-nine minutes, then part of it will be eclipsed.

Although not explicitly stated, it is clear from the figures that a necessary and sufficient condition that totality be attained is that

$$r_s - r_m > \beta.$$

The numbers in the Nuzhah lead to the equations

$$r_s + r_m = 63'$$
$$r_s - r_m = 29',$$

whose solution is $r_s = 46'$ and $r_m = 17'$, both numbers being rounded-off Ptolemaic parameters ([43], ed. of Halma, vol.i, p.395).

That the latitude condition for a partial eclipse is equivalent to the statement made in our text about the ecliptic distance from node to opposition may be demonstrated as follows.

Consider the right spherical triangle (Figure 18) formed on the celestial sphere by the node, the moon's center, and the projec-

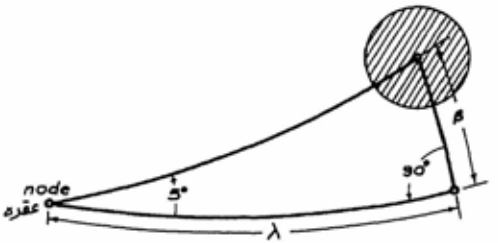

Figure 18. The Lunar Latitude Near Syzygy

tion of the latter on the ecliptic. The relation

$$\sin \lambda \approx \frac{\beta}{5^\circ}$$

subsists, since β and 5° are both small. Putting $\beta = 63'$ we obtain

$$\lambda = \text{arc sin } (63/300) \approx 12^\circ,$$

as demanded by the text.

We are now in a position to investigate the condition with which Chapter II,11 opens, namely that if the opposition occurs during daylight it must be less than 2;4 hours after sunrise or before sunset. Violation of the condition implies that there will be no possibility of any part of even the longest eclipse taking place while the sun is below the horizon.

An eclipse of maximum duration is one for which first contact takes place when the elongation is sixty-three minutes of arc.

Since the mean rate of elongation is about 13;10,15° - 0;59,8° = 12;11,27° per day, half the length of the longest eclipse is about

$$\frac{1;3(24)}{12;11,27} = 2;4 \text{ hours.}$$

This is doubtless the origin of the number the author gives. It does not appear in the Nuzhah.

36. Lunar Eclipse Markings on the Ruler

A permanent mark (O in Figure 19) is placed on the ruler's edge at a distance of sixty-three parts (sixtieths of the plate radius) from the end. The mark of first totality (O") is put permanently on the ruler at a distance twenty-three parts from the same end. The segment OO" will be thirty-four parts in length, equal to the apparent diameter of the moon in minutes. It is divided into twelve equal segments, each one an eclipse digit.

37. Determination of the Lunar Eclipse Times

Suppose now that β for the time (t) of the opposition has been computed and is sufficiently small to indicate an eclipse. Set the alidade point of Aries on the ring. Put a mark O on the plate such that the segment DO is of length β measured in divisions of the alidade, taking a sixtieth of the plate radius for each minute of arc. Rotate the alidade through an angle of ninety degrees, and place the ruler so that the lunar eclipse mark on its edge coincides with O, and so that its end touches the edge of the alidade (at M). The triangle DOM is now a mechanical construction of the triangle having the same letters in Figure 16. Hence the length of DM on the instrument gives

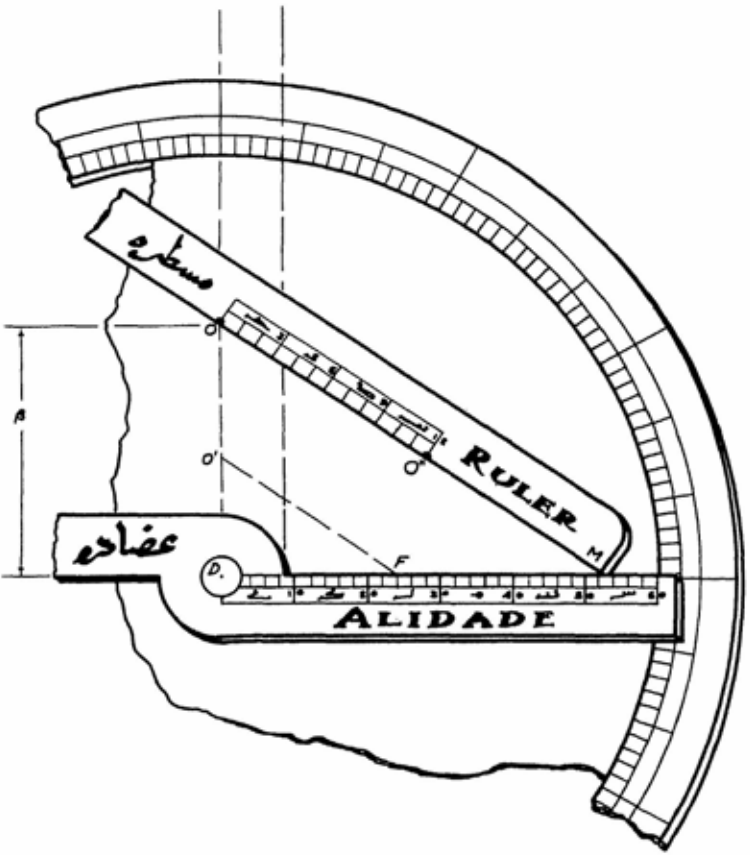

Figure 19. The Instrument Set Up for a Lunar Eclipse Determination

the elongation in minutes of arc at the instant of first (or last) contact. Since the rate of elongation is, very crudely, a half degree per hour, "depressing" (cf. Section 3) the length

of DM once and doubling the result (i.e. dividing by a half) will yield the elapsed time from first contact to the middle of the eclipse. This is what the text prescribes. Add the result to t and subtract it from t to get the times of day at which last and first contact take place respectively.

If the eclipse is total, an exactly analogous use is made of the mark of first totality (O") instead of the lunar eclipse mark (O) to construct the triangle O'FD (in Figure 19) congruent with OFD (in Figure 17). From it is obtained DM, the elongation at first totality, which also is converted into time by doubling and depressing. Addition to and subtraction of the result from t gives the times of day of last and first totality respectively.

For a partial eclipse the magnitude is measured very simply by sliding the end (M) of the ruler along the alidade until the ruler occupies the position indicated by the pair of dotted lines on Figure 19. The position of the mark O with respect to the digit scale on the ruler then gives the magnitude in digits directly. The validity of this procedure can be seen from Figure 16, where it will be observed that the eclipse magnitude is, approximately

$$EC = ED + DC = r_m + r_s - \beta = 63' - \beta,$$

which is the construction prescribed.

38. Parallax in the Altitude Circle (f. 35r:4 - 36r:5)

It is convenient here to break the order of the text and to insert appropriate comment on Chapter II,14, the substance of which is a preliminary to Section 39.

In Figure 20 assume the small circle with center C to be the terrestrial sphere. An observer at E takes the altitude of

a celestial body at S, the latter being at a distance SC from the center of the earth. With respect to the observer's

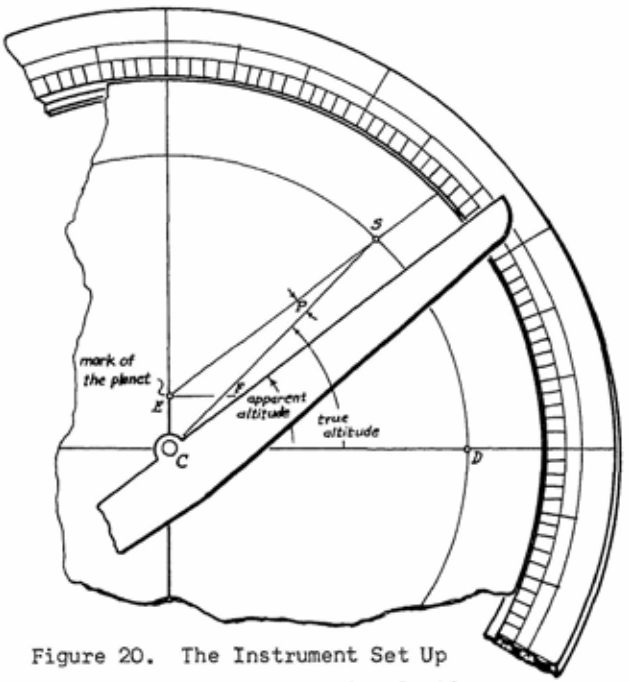

Figure 20. The Instrument Set Up for a Parallax Determination

horizon, the tangent to the sphere at E, the altitude of S is angle SEF. As reckoned from the center of the earth, however, the altitude is SCD. The difference between these two angles is P, the parallax in the altitude circle. It is clear that P is a function of two variables, the earth-planet distance, and the altitude. For CS constant, P is maximum when the altitude is zero, taking a maximum value of, very nearly

$$\text{arc sin } \frac{EC}{CD} .$$

The operation described in Chapter II,14 consists essentially of making a scale representation of the situation set forth above, and our Figure 20 is sufficient supplement to the author's explanation.

It is of interest, however, to put down his parameters. For the three objects mentioned, horizontal parallax at mean distance will be

☾	arc sin 0;1,2 = 0;59,13°
☉	arc sin 0;0,2 = 0; 1,55°
♀	arc sin 0;0,4 = 0; 3,50°,

where mean distance has been taken as 1,0;0 for the sun and moon and as 1,0;0/2 for Venus, as prescribed in f. 35r:13.

Comparison of these numbers with corresponding entries in a table of parallaxes on f. 185v of Kāshī's zīj [23] show as close a correlation as can be expected. Horizontal lunar parallax at mean distance is there given as 0;59,59°. For the sun it is 0;2,15°, and this accords with our result of 0;1,55°, since the author says vaguely (f. 35r:9) that for the sun CE should be "a little more" than the two minutes we have taken. For Venus the zīj gives 0;4,8°, likewise reasonably close to our value.

For both the sun and Venus the constructions prescribed bear little relation to practical reality. For one thing, the measurement of angles as small as these on an instrument of this sort is out of the question. And for another, the distance of Venus from the earth is based on no observations at all, but on the assumption that the planetary system has been put together

by nesting the successive Ptolemaic configurations as closely inside each other as will insure no planet's striking its neighbor.

39. The Table of Parallax Components (f. 33v)

In order that a solar eclipse be visible to an observer it is necessary that the sun and moon have, for the time of the eclipse, very nearly the same celestial coordinates. And since the observer is on the earth's surface, not at its center, it is clear that the coordinates used must be apparent ones and not true coordinates, that is, computed with respect to the observer rather than with respect to the center of the earth. In other words, it is necessary that a correction for parallax be applied to the true coordinates resulting from ordinary determinations.

The adjusted lunar parallax (ikhtilāf-i manẓar-i muʿaddal-i qamar) is the difference between the parallax of the sun and moon at a time when both have the same apparent altitude. Our table breaks up this adjusted parallax in the altitude circle into its latitude and longitude components. Each pair of such components is for a conjunction occurring at an integer number of hours before, after, or at the local meridian, and at the initial point of one of the twelve zodiacal signs. Symmetry considerations permit the use of a single column for each pair of signs equidistant from the solstices; thus the second column from the left (on the transcription) serves for both Leo and Gemini, provided that the hours for the former are read from the column at the extreme right, and those for the latter from the left. The latitude components are given in minutes of arc; the longitude entries in minutes of time by which the effect of the parallax will delay or advance the time of apparent conjunction.

Since the rate of elongation is about twelve degrees per day, division of the longitude components by two gives an approximate conversion into minutes of arc.

The top row of entries in the table gives the length of daylight when the sun is in the beginning of the sign named at the head of each respective column.

Our author's statement (f. 14r:7) that the table is from Kāshī's zīj is confirmed by examination of the latter document. It is on f. 164v of [23], where the statement is made that the data have been computed for a place of terrestrial latitude 30°. In a special study [27] it has been shown that the numbers in the table are only crudely correct, and that they were probably lifted by Kāshī from some source unknown to us.

40. Solar Eclipses (f. 33r:5 - 34v:3)

The technique for computing solar eclipses with the aid of the instrument closely resembles that used for lunar eclipses, and can be described with reference to what has preceded. Of course it is now conjunctions, not oppositions, which present possibilities for solar eclipses. Just as before, a permanent mark is put on the edge of the ruler, this time at a distance from the end of thirty-three sixtieths of the plate radius. (Cf. f. 12v:7). This implies that the (apparent) lunar latitude at conjunction must be less than thirty-three minutes, a condition which the text gives explicitly in f. 34r:9. The same limit is given in the Almagest ([43] vi,5). The condition also implies that

$$r_m + r_\sigma = 33'$$

where r_σ denotes the apparent radius of the solar disk. Since

we have already taken r_m to be about seventeen minutes, it follows that the solar and lunar disks are assumed to be of about the same size. Hence even if a solar eclipse is total, the duration of totality will be very short, and in fact no mark of first totality is prescribed. The portion of the ruler edge from the solar eclipse mark to the end is to be divided into twelve equal parts for the solar eclipse digits (f. 13r:2).

One complication which does not enter in the case of a lunar eclipse is the fact that for a solar eclipse the geographical location of the observer is involved. Hence the true coordinates are to be corrected for parallax by use of the table on f. 33v. The effect of parallax being always to depress the apparent position beneath the true one, it follows that if the conjunction occurs in the forenoon the (longitudinal) correction will be subtracted from the time of true conjunction, whereas it is added in the case of an afternoon conjunction. For the same reason the latitude correction is subtracted algebraically from the true latitude, north being taken as positive. Once the corrections have been made, the time from first contact to the middle of the eclipse and the magnitude of the eclipse are computed on the instrument just as they are for a lunar eclipse.

The parallax also acts to asymmetrize the necessary condition for a solar eclipse. Its effect is always to pull a celestial object down along a vertical circle from its true position. Hence when the true moon is north of the ecliptic, the allowable distance between node and conjunction is greater than when it is south. Thus the chapter on solar eclipses in the Nuzha (NS, p.281) begins with the statement that if the conjunction is after the ascending node and before the descending node, i.e. if the moon has north latitude, the critical distance is

sixteen degrees; if the moon has south latitude the distance is seven degrees.

The corresponding passage in our text, f. 33r:7-12, doubtless said the same thing in the original composition. However, it was garbled by the copyist who wrote twice the line and a half enclosed in braces in the translation. His mistake was made easier by the fact that the words for distance (bu^{c}d) and after (ba^{c}d) are written the same unless diacritical marks are used.

In his zīj ([23], f. 85r) Kāshī gives the same necessary condition as is found in our text and in the Nuzhah. He qualifies it by saying that it applies to localities in the third and fourth climates. A criterion which holds for all inhabited localities is, he says, that if the lunar latitude is north, the distance from node to conjunction shall be less than eighteen degrees; if south the distance shall be less than nine degrees. He does not derive this condition in the zīj, and it may very well be rounded off from Ptolemy's 17;41° and 8;22° arrived at in the Almagest ([43], vi,5).

41. The Solar Mean Longitude at Equinox (f. 34v:4 - 35r:3)

In Islamic astrology the instant at which the sun crosses the vernal equinoctial point is called the year-transfer (Arabic tahwīl al-sinah, or simply tahwīl, cf. [4], p.150) and was considered to be of great significance. Even to the present in Iran the situation of the individual at the moment of transfer is supposed to affect his destiny throughout the coming year.

Chapter II,13 in our text explains a method for determining the time of transfer by use of the equatorium. The same problem is treated much more elaborately in the Khāqānī Zīj ([23], ff. 90r - 91r). It can be formulated as the inverse of a more

common problem, the determination of a planetary true longitude (λ) as a function of its mean longitude ($\bar{\lambda}$), the latter being a linear function of time. Now we have the solar true longitude given, at the equinoctial point ($\lambda = 0^o$), and we seek the corresponding $\bar{\lambda}$. Once this is determined the corresponding time can be inferred from the mean motion table.

The solutions in the zīj are set up in terms of the mean and true centers, $\bar{\lambda}_a$ and λ_a, but the difference is trivial, since addition of the apsidal longitude to a center converts the latter into a longitude. (Cf. p.188.)

Two apparently alternative methods are given in the zīj for obtaining $\bar{\lambda}_a$ from λ_a. The first says, find from the table of solar equations (e) the equation whose argument is the given λ_a. Add the result to λ_a to obtain a first approximation to $\bar{\lambda}_a$, and repeat the process. That is, put

$$\bar{\lambda}_{a1} = e(\lambda_a) + \lambda_a,$$
$$\bar{\lambda}_{a2} = e(\bar{\lambda}_{a1}) + \lambda_a,$$
$$\bar{\lambda}_{a3} = e(\bar{\lambda}_{a2}) + \lambda_a.$$

Then, says Kāshī, $\bar{\lambda}_{a3}$ is the desired $\bar{\lambda}_a$. This iterative method for inverting suitable types of functions is very old, and probably of Hindu origin (cf. [1] and [32]).

The second method consists of putting

$$\text{Sin } e(\lambda_a) = \sin e_{max} \cdot \text{Sin } \lambda_a,$$

where $e_{max} = 2;0,29^o$ is Kāshī's maximum solar equation. Then

(24) $$\bar{\lambda}_a = e(\lambda_a) + \lambda_a = \text{arc Sin}(\sin e_{max} \cdot \text{Sin } \lambda_a) + \lambda_a.$$

In another part of the zīj, on f. 166v, is a numerical table of the function $L(\lambda_a)$, say,

$$L(\lambda_a) = 2;0,29^\circ \sin \lambda_a + \lambda_a. \tag{25}$$

e_{max} being a small angle, expressions (24) and (25) are approximately equivalent. Thus the table gives a third method for obtaining $\bar{\lambda}_a$.

By contrast with these elaborate and approximate techniques, the solution with the instrument is direct and, at least theoretically, precise. One puts the ruler alongside the fictitious center and parallel to the line joining the plate center and Aries 0°. The point of intersection of the ruler edge with the graduations of the ring gives immediately the mean longitude at which $\lambda = 0^\circ$, the configuration being the same as that for determining λ from $\bar{\lambda}$.

The desired $\bar{\lambda}$ having been determined, it is easy to find from the table of mean motions a date such that at the noon of that day the solar mean longitude $\bar{\lambda}_n$ exceeds the equinoctial $\bar{\lambda}$ just found, whereas on the preceding noon the solar mean longitude is exceeded by the equinoctial $\bar{\lambda}$. The expression $(\bar{\lambda}_n - \bar{\lambda})/\dot{\bar{\lambda}}$ then gives the number of hours from the instant of transfer to the later noon, provided that $\dot{\bar{\lambda}}$ is the rate of change of $\bar{\lambda}$ in degrees per hour.

For material on f. 35r:4 - 36r:5 see Section 38 above

42. The Equation of Time (f. 36r:6 - 37r:8)

Chapter II,15 of the text contains no application of the equatorium, and in fact makes no reference to either one of our instruments. The topic it discusses is, however, of at least

theoretical interest in the major problem solvable with the equatorium, the determination of planetary true positions. The table of mean motions, on which such determinations are ultimately based, gives mean positions reckoned from some epoch. Local apparent time, reckoned from successive meridian passages of the true sun, differs from mean time because of two facts. These, as the author points out, are the variable angular velocity of the true sun in the ecliptic, and the unequal projections on the celestial equator of equal segments on the ecliptic. The difference is the equation of time.

The first part of the chapter is copied verbatim from the extensive equation of time material in the Khāqānī Zīj ([23], f. 93r). At about f. 36v:10 our author abandons this source, and from there on leans heavily on Appendix 8 (p.311) of NS. The NK version has nothing on the equation of time; the subject is another afterthought of Kāshī, but his presentation is much more satisfactory than that of our text, which slavishly copies his mistakes.

For instance, f. 37r:3-37r:6 in the text is on the conversion from degrees of arc to time, 360° being equivalent to twenty-four hours. A degree of arc does correspond to four minutes of time, and a minute of arc to four seconds, but ten minutes of arc corresponds to forty seconds of time, not to a minute, as both the NS and our text have it.

In order that this chapter be useful the author should have provided the user with specific means of determining the equation of time. One way would be to give a numerical table such as that in Kāshī's own Khāqānī Zīj ([23], ff. 126v, 127r). Another way would be to give explicit directions for computing both components of the equation of time. The determination of one component, the solar equation, has already been described,

in Chapter II,6. Nothing has been done in the text, however, about the computation of the right ascensions which make up the second component. In NS (p.311) Kāshī describes a technique with the instrument for solving this spherical trigonometric problem. The methods resemble those used for the determination of the lunar latitude and commented on in Section 25 above.

To clarify the passage f. 36v:11 - 37r:2 we remark that the equation of time (E) at any instant t is the difference between the change of mean solar longitude ($\bar{\lambda}$) from epoch (t_0) to time t, and the change in right ascension (α) of the true sun from t_0 to t. Symbolically this is

$$\begin{aligned} E(t) &= \Delta\bar{\lambda}(t) - \Delta\alpha(t) \\ &= [\bar{\lambda}(t) - \bar{\lambda}(t_0)] - (\alpha(t) - \alpha(t_0)) \\ &= \bar{\lambda}(t) - \alpha(t) - \bar{\lambda}(t_0) + \alpha(t_0). \end{aligned}$$

The rule in the text says

$$E(t) = (\bar{\lambda}(t) + 3;57,30^\circ) - \alpha(t).$$

Comparison of the two expressions shows that we must have

$$\alpha(t_0) - \bar{\lambda}(t_0) = 3;57,30^\circ,$$

that is, the constant to be added to the mean longitude is a number which depends on the epoch of the tables and on the base longitude for which they have been computed. This $3;57,30^\circ$, however, has been taken over from NS without change, whereas the mean motion tables of the text have both a different epoch and a different base longitude than those of NS.

43. The Plate of Conjunctions (f. 37r:9 - 38r:11)

The construction of the second instrument is easily understood from the description on ff. 15r-17r, especially when considered in conjunction with the drawing on f. 17v, its modern counterpart on the opposite page, and Figure 21, reproduced from page 287 of NS.

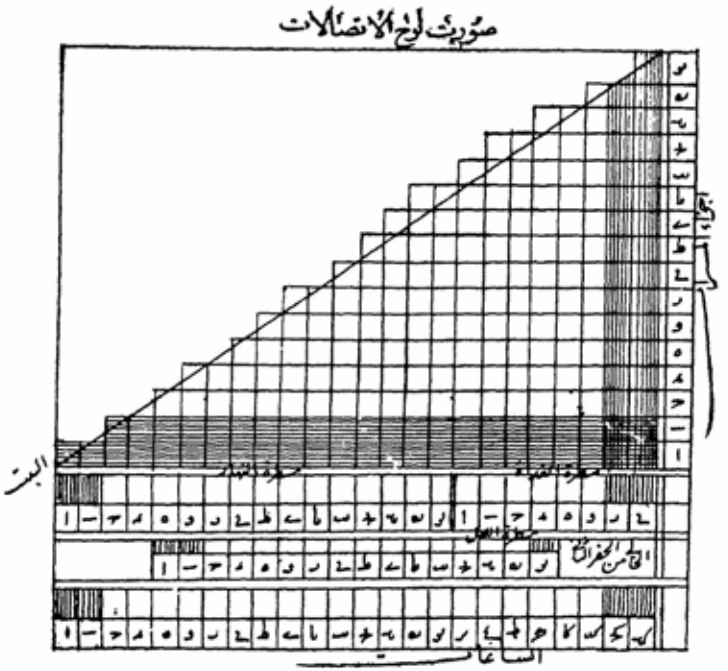

Figure 21. The Plate of Conjunctions, from NS

Of the metrological units mentioned in this passage, the size of the cubit has been discussed in Section 2 above. The

digit is usually taken as one twenty-fourth of a cubit.

As for the operation of the instrument, the device has been designed to predict the time of day at which a conjunction between two planets will occur, given the noon longitudes of the planets for the given day and for the following day. It is further assumed, of course, that their longitudes show one planet in the lead at the first noon, and the other at the second. Regard the variations in longitude as linear throughout the course of the day. Then if t is the time in hours from the first noon until the conjunction takes place,

$$t = \frac{24d}{b} ,$$

where d is the difference between the longitudes of the two planets (Kāshī's past distance), and b is the difference between the daily rates of the two planets (Kāshī's daily motion) for the day in question. Incidentally, the Persian-Arabic term buht, standard for the longitudinal speed of a planet, is from the Sanscrit word bhukti (cf. [4], p.105).

The turning ruler (f. 17r:12, see also Figure 2) is set so that its edge crosses the "divisions of travel" scale at distance b from the beginning of the scale. Find the point on the same scale corresponding to d, from there project horizontally to the edge of the turning ruler, thence vertically down to the base of the triangle. It is clear that the resulting point is at the required distance t from the left vertex of the triangle.

The three sliding scales at the bottom are to convert t from time measured from noon to time measured from local sunrise or sunset, depending on whether the event occurs during the day or the night respectively. To do this, set the day ruler so

that the pivot on the turning ruler is opposite the point on the day ruler corresponding to half the length of daylight for the date and locality in question. Set the head of the night ruler opposite the point on the day ruler corresponding to the number of hours of daylight. Put the head of the next-day ruler opposite the point on the night ruler corresponding to the number of hours of darkness. Then, as the author remarks, the right angle of the triangle will fall opposite the point of the next-day scale marking the hour of noon. Now the place where the vertical line distant t units from the pivot intersects one of the three slides at the base gives directly the desired hour of day or night.

In all of the late medieval Persian zījes inspected by the editor, many pages are given over to a double-entry table of the function defining t above (cf. [29], p.162). It is evident that Kāshī invented this simple device to obviate the need for such tables. As such it fulfilled a practical purpose, yielding results of sufficient precision for the problem at hand. The instrument's most serious drawback follows from the fact that the usual daily motion of the planets is of the order of a degree. That of the moon is much larger, averaging over thirteen degrees. This implies that if the conjunction does not involve the moon, the turning ruler would be elevated by so small an angle that the result would be considerably affected by small inaccuracies in the construction.

PERSIAN GLOSSARY

(Readers who desire to locate in the text one of the words listed below can do so by looking up its English equivalent in the index.)

ا

آلت	instrument
اتصال (pl.) اتصالات	conjunction(s)
اثقاب	see: ثقب
اجتماع	conjunction
اجزاء	see: جزء
اختلاف	difference
اختلاف منظر	parallax
ارتفاع	altitude
ارقام اختلاف	difference marks
استقامت	forward (motion)
(مقام) استقامت	direct (station)
استقبال	opposition
اسطرلاب	astrolabe
اصابع	digits
اقسام	see: قسم
اقليم (pl.) اقاليم	climate(s)
الوان	see: لون
انجلاء	clearance (of an eclipse)
انحراف	obliquity
انقلابين	solstices
اهليلجي	elliptical (?)
اوج (pl.) اوجات	apogee(s)
اوساط	mean (motions and positions) see: وسط

ب

بدل	substitute
برج (pl.) بروج	sign(s) (zodiacal)
بطي	slow
بُعد (pl.) ابعاد	distance(s)
بُعد مضاعف	doubled distance
بهت	rate

پ

پرگار	compass

ت

calendar or date	تاریخ
complete (of years)	تامہ
transfer	تحویل
epicycle	تدویر
doubling	تضعیف
equation	تعدیل
equation of time	تعدیل الایام بلیالیہا
differences	تفاضل
subtraction	تفریق
intersection	تقاطع
true (celestial) longitude	تقویم
halving	تنصیف
succession of the signs	توالی بروج

ث

hole, drill hole	ثقب

ج

table(s)	جدول (pl.) جداول
part(s), or division(s)	جزء (pl.) اجزاء
addition	جمع
south	جنوب
node, lunar	جوزہر
sine	جیب

ح

acute (angle)	حادہ
product	حاصل ضرب
ring	حجرہ
motion(s)	حرکت (pl.) حرکات
computation	حساب
argument	حصہ
perigee	حضیض
depression, or trough	حفر
true	حقیقی
ring	حلقہ
Aries	حمل

خ

quotient	خارج قسمت
eccentric	خارج مرکز
anomaly	خاصہ
compound anomaly	خاصہ مرکبہ
lunar eclipse	خسوف
wood	خشب

PERSIAN GLOSSARY

ز

angle	زاویه
right angle	زاویه قایمه
increasing	زاید
projection, excess, lug	زائده
tongue	زبانه
Saturn	زحل
Venus	زهره
astronomical handbook	زیج

س

year	سال
complete (or elapsed) year	سال تامه
speedy, fast	سریع
plane	سطح مستوی
the two inferior planets, Mercury and Venus	سفلیین
immersion (of an eclipse)	سقوط
chain	سلسله
year(s)	سنه (pl.) سنین
hole	سوراخ
movement	سیر

ش

line(s)	خطّ (pl.) خطوط
line of apsides	خط اوجی
latitude lines	خطوط عرض
thread	خیط

د

circle(s)	دایره (pl.) دوایر
latitude circle	دایره عرض
degree(s)	درجه (pl.) درجات
rotation, or revolution	دور

ذ

cubit	ذراع
epicyclic apogee	ذروه

ر

retrograde	راجع
quadrant(s)	ربع (pl.) ارباع
retrogradation, or retrograde	رجعت
elevate (a sexagesimal), to	رفع کردن
numeral(s), or mark(s)	رقم (pl.) ارقام
difference mark	رقم اختلاف
string	ریسمان

alidade	عضاده
Mercury	عطارد
magnitude (of a star)	عظم
node	عقده
mark	علامت
mathematics	علم ریاضی
astronomy	علم نجوم
superior (planets)	علویه

غ

extremity, extreme (value), maximum	غایت

ف

Persian	فارسی
excess, or difference	فضل
ecliptic, or zodiac	فلک البروج

ق

base	قاعده
perpendicular	قایم
qibla, the direction of Mecca	قبله
disk	قرص
Constantinople	قسطنطنیه

ش

plumbline	شاقول
day, nychthemeron	شبانه روز
yellow copper	شبه
sun	شمس

ص

brass	صفر
plate	صفیحه

ض

multiplication	ضرب

ط

plate of the heavens	طبق المناطق
longitude, usually terrestrial longitude, but in the text (f.3r:5) celestial longitude.	طول
longitudinal	طولی

ظ

shadow	ظل

ع

universe	عالم
latitude(s)	عرض (pl.) عروض
bride, (theorem of the)	العروس

sine (astrolabe or quadrant) — جیب

preserved — محفوظ

axis — محور

circumference — محیط

orbit, on f.8r:11, but in an astronomical context the word usually denotes any circle of the celestial sphere whose pole is the north pole. — مدار

square — مربع

place, the place of a digit in a place-value representation of a number. — مرتبه

elevated — مرفوع

compound (in compound anomaly). — مرکبه

center(s), sometimes mean longitude measured from deferent apogee. — مرکز, مراکز (pl.)

turning center — مرکز مدیر

fictitious center — مرکز مستعار

apparent — مرئی

pointer — مری

Mars — مریخ

fictitious — مستعار

direct (motion), of a planet — مستقیم

division(s), or part(s) — قسم, اقسام (pl.)

diameter — قطر

equating diameter — قطر استواء

moon — قمر

pivot, pole — قطب

arc(s) — قوس, قسی (pl.)

ك

fractions — کسور

solar eclipse — کسوف

star(s), or planet(s) — کوکب, کواکب (pl.)

total (of an eclipse) — کلی

ل

sights — لبنتین

plate — لوح

color(s) — لون, الوان (pl.)

م

inclining, or inclined — مایل

explicit (years) — مبسوطه

triangle — مثلث

sum — مجموع

ruler	مسطره
peg	مسمار
path(s)	مسیر (pl.) مسیرات
Jupiter	مشتری
ascensions, or risings	مطالع
adjusted	معدّل
equant	معدّل المسیر (or) معدّل للمسیر
equator, celestial	معدالنهار
divisions	مقاسم
station(s)	مقام (pl.) مقامات
true position	مقوم
stationary, stance	مقیم
duration (of an eclipse), first totality	مکث
tangent	مماس
parecliptic	ممثّل
see: منطقه	مناطق
depress (a sexagesimal), to	منحط کردن
heaven(s), deferent(s)	منطقه (pl.) مناطق
parallel	موازی
position	موضع
Libra	میزان
inclination, or declination	میل

ن

decreasing	ناقص
incomplete (of years), or current, or explicit	ناقصه
copper	نحاس
ratio	نسبت
noon, or meridian	نصف النهار
semicircle	نصف دایره
sector(s)	نطاق (pl.) نطاقات
opposite	نظیر
decrease	نقص
point(s)	نقطه (pl.) نقاط
latitude point	نقطه عرض
opposite point	نقطه محاذات
light	نور
luminaries, the two, the sun and the moon)	نیرین

و

chord, or hypotenuse	وتر
mean (position or motion)	وسط (pl.) اوساط

ے

Yazdigerd	یزدجرد

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